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HOW FAR CAN GET FRB PROGENITOR NEUTRON STARS FROM THEIR BIRTHPLACE?

The recent studies show evidence of magnetars – young neutron stars being good candidates of fast radio birth sources. Neutron stars can form as remnants of type II supernovae explosions of young stars. That is, such young neutron stars should be associated with their parent young star cluster. We study how these FRB source candidates can get far out of their birthplace if they escape clusters due to their high kick velocity. Therefore, we perform numerical simulations of the early evolution of star clusters and trace all young neutron stars. We found that neutron stars can remain in clusters, as well as escape them during their possible magnetar phases. There is no preference in time during the cluster evolution for FRBs to happen in or out of the parent cluster. The maximum distance the candidate FRB progenitors could escape from the cluster does not exceed 250 pc. That is, the high dispersion measure of FRBs might be influenced by the expulsed ionized residual star-forming gas in the parent molecular cloud.

Key words: fast radio burst progenitor, young star cluster, young neutron stars.

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Лездік радио жарқылдарын тудырушы нейтронды жұлдыздар туған өлкесінен қаншалықты алысқа кете алады?

Соңғы зерттеулер бойынша магнетарлар – жас нейтрондық жұлдыздар – лездік радио жарқылдарының көздеріне жақсы үміткерлер екенін көрсетті. Нейтрондық жұлдыздар II типті аса жаңалардың жарылыстарының қалдықтары ретінде пайда бола алатыны белгілі. Яғни, мұндай жас нейтрондық жұлдыздар өздерінің аналық жас жұлдызды шоғырмен байланысты болуы керек. Бұл жұмыста жарылыстан кейінгі қосымша жылдамдыққа ие болып, шоғырдан шығып кететін лездік радио жарқылдарға үміткерлер туған өлкесінен қаншалықты алыстай алатындығы қарастырылады. Сондықтан, біз жұлдыздық кластерлердің ерте эволюциясын сандық модельдеу жүргіземіз және барлық жас нейтрондық жұлдыздарды қадағалаймыз. Нейтрондық жұлдыздар болжалды магнетар күйінде болған кезде шоғырда болуы да, одан шығып кетуі мүмкін екендігі анықталды. Шоғырдың эволюциясы барысында, уақытқа тәуелсіз, лездік радио жарқылдар өз кластерінің ішінде де, одан тыс жерлерде де болулары мүмкіндігі көрсетілді. Лездік радио жарқыл көзінің үміткерлері кластерден алыстай алатын максималды қашықтық 250 пк-тен аспайтыны анықталды. Яғни, аналық молекулалық бұлттағы иондалған, қалдық жұлдыз түзуші газ лездік радио жарқылдардың дисперсия өлшемдерінің жоғары болуына өз әсерін тигізуі ықтимал.

Түйін сөздер: лездік радио жарқылдардың ықтималды көздері, жас жұлдыздық шоғырлар, жас нейтрондық жұлдыздар.

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Как далеко могут уйти нейтронные звезды как возможные источники быстрых радиовсплесков от места формирования?

Недавние исследования показали, что магнетары – молодые нейтронные звезды – являются хорошими кандидатами на источники быстрых радиовсплесков. Нейтронные звезды могут образовываться как остатки взрывов сверхновых звезд II типа. То есть, такие молодые нейтронные звезды должны быть связаны со своим родительским молодым звездным скоплением. В данной работе рассматривается, как кандидаты на быстрые радиовсплески могут удаляться далеко от места своего рождения, если из-за высокой скорости убегания они покинут скопление. Поэтому мы проводим численное моделирование ранней эволюции звездных скоплений и отслеживаем все молодые нейтронные звезды. Было обнаружено, что нейтронные звезды могут оставаться в скоплениях, а также покидать их, пока они находятся в возможном состоянии магнетара. Во время эволюции кластера, независимо от времени, быстрые радиовсплески происходят в родительском кластере или за его пределами. Максимальное расстояние, на которое кандидаты на быстрые радиовсплески могли удалиться от кластера, не превышает 250 пк. То есть, на высокий показатель дисперсии быстрых радиовсплесков может влиять выброшенный ионизированный остаточный звездообразующий газ в родительском молекулярном облаке.

Ключевые слова: источники быстрых радиовсплесков, молодое звездное скопление, молодые нейтронные звезды.

Introduction

Fast radio bursts (FRB) are known as millisecond duration radio signals from unknown sources almost all of extragalactic origin [1]. The phenomena of FRBs remained a mystery since they were found first in 2007 by Lorimer et al [2]. The majority of FRB signals were observed only once. However, there are repeating FRBs observed too [3,4]. Some of the repeating FRB sources were localized to a few distant galaxies [5,6,7,8]. However, the recent discovery of FRB signal from the Galactic source, associated with the soft-gamma repeater magnetar SGR1935+2154 [9,10] has opened a window for our understanding of the mystery of FRBs. Magnetars – young neutron stars with a very strong magnetic field have been a strong candidate for FRB source at a later time [11,12]. The new discovery of the FRB signal from the known Galactic magnetar has strengthened this hypothesis even further [13]. It is believed that some magnetars originate from the core-collapse supernovae (SNe) [14]. All Galactic magnetars have been observed to be in the age range of 10^3-10^5 yr [15,16]. Therefore, in this work, we perform N-body simulations of the formation and early evolution of galactic star clusters with different masses and star-formation efficiencies. Then we study the distribution of young neutron stars in the young star clusters as candidates for FRB progenitors. And determine the maximum distance of young neutron stars from their parent clusters as possible FRB progenitors.

Methods and simulations

Star clusters form from dense clumps of gas in molecular clouds [17]. Once massive O-B stars form their feedback to the ambient gas in the form of ionizing radiation, stellar wind and radiation pressure cause the gas expulsion [18]. Star clusters with different star-formation efficiencies (SFE) evolve differently after gas expulsion, some of them dissolving during the violent relaxation – the dynamical response to gas expulsion [19,20]. Therefore, considering the violent relaxation and star-formation efficiencies one can account for the formation conditions of star clusters, thus making the simulation more complete.

In this study, we consider model clusters similar to S0-models of Shukirgaliyev et. al [21]. To be more precise we use their initial conditions for model clusters with masses $M_* = 3000M_\odot$, $M_* = 6000M_\odot$ and SFE = 0.15, 0.17, and 0.25, three randomizations each. Series of works [20,21,22] studied the evolution of star clusters formed with a centrally peaked star-formation efficiency profile according to the model of Parmentier & Pfalzner [23] after instantaneous gas expulsion. S0-models of [21] are star clusters orbiting in the Solar Neighborhood (i.e., circular orbit with a radius of 8 kpc in the Galactic disk plane).

Shukirgaliyev et al [20] first considered clusters formed with centrally-peaked SFE-profile and found that such clusters resist better to the instantaneous gas expulsion than clusters formed with radially constant SFE [19]. Also, such star clusters evolve during the violent relaxation independent of the cluster initial mass and the impact of the Galactic tidal field [21]. Shukirgaliyev et al [22] have discovered that star clusters formed with low SFE dissolve faster than those formed with high SFE, even if they have the same masses.

In contrast to models of [21] in our simulations, we include the natal kick of SNe remnants and additional information output about young NSs while the latter is still in the phase of magnetar activity. We assume that the magnetar phase of neutron stars starts 10^3 yr after SNe and lasts for 10^5 yrs [16]. The natal kick velocities of SNe remnants are distributed by Maxwell distribution with $\sigma = 265$ *km/s* [24].

In N-body codes, the output period usually scales with the dynamical time, which is about a Myr for Galactic open clusters [25]. However, the timescale of magnetar lifetime is very short compared to the open cluster dynamical times. Therefore, we have modified our direct N-body code to catch at least a few moments when a neutron star remains in the magnetar phase. The phi-GRAPE/GPU code [26,27] has been modified in such a way that it now also gives additional output information about young neutron stars with a high-frequency output (almost every few thousand years).

Our simulations start at the time of instantaneous gas expulsion and continue for about 90 Myr when all neutron stars formed from core-collapse SNe become too old to be a magnetar. In total we performed 18 simulations: 9 simulations with cluster initial mass of $M_* = 3000M_\odot$, where we consider 3 random realizations per SFE (0.15, 0.17, 0.25) and 9 simulations with cluster mass of $M_* = 6000M_\odot$. We selected SFE=0.15 as it represents clusters surviving the gas expulsion with a very small bound fraction (about 7 percent), thus they are the most expanding clusters. Star clusters with SFE=0.25 survive the gas expulsion with a mean bound fraction of about 50-60 percent and represent the long-living clusters [22]. SFE=0.17 is something in between the two mentioned above and represents the middle case.

Results

Distribution of possible FRB candidates around young clusters. Our model clusters are in a super-virial dynamical state at the beginning of simulations, i.e., at the time of instantaneous gas expulsion. The lower the global SFE, the higher the initial virial ratio of the cluster. Therefore, our model clusters start expanding right after gas expulsion and some massive stars leave the cluster before they experience SNe explosion. Three panels of Fig 1. present the distribution of possible FRB candidates (i.e., possible magnetars) in space (i.e., the distance from the cluster center) and time (i.e., starting and ending of possible magnetar phases, X-axis, where ending points are made slightly transparent) around model clusters with $M_* = 3000M_\odot$. Solid lines show the Jacobi radii of clusters through time. Different colors correspond to different random realizations and each panel represents models with given SFE.

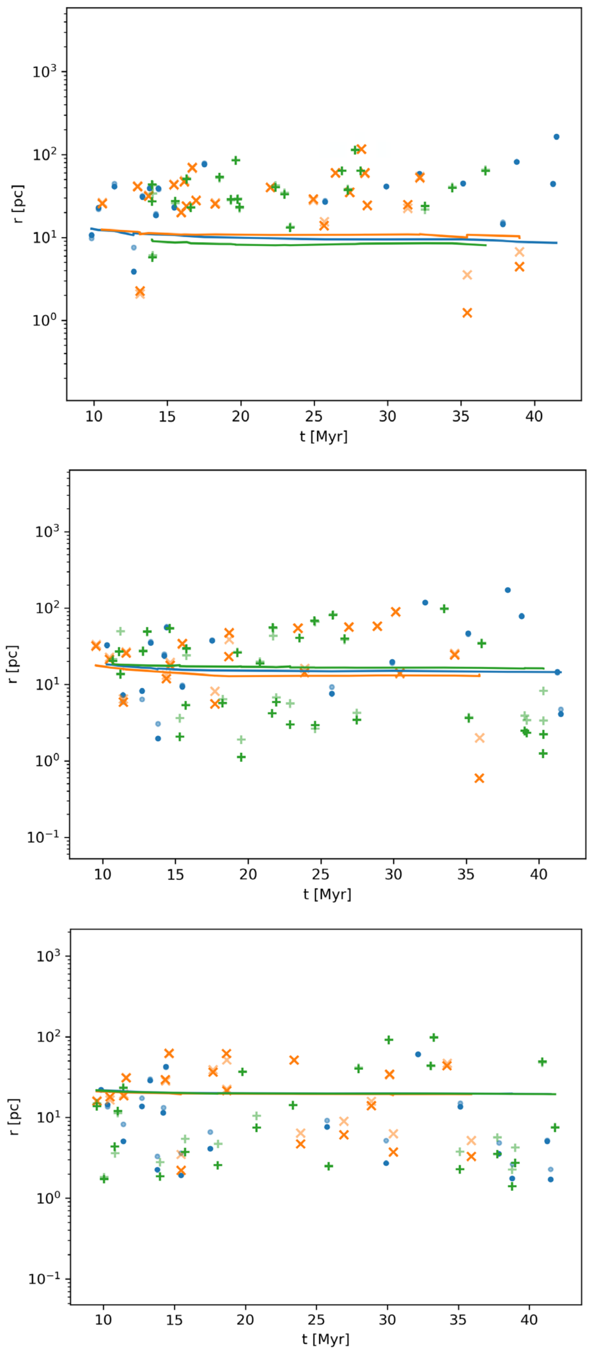

Figure 1 – Distribution of possible FRB candidates around clusters with masses of $M_* = 3000M_\odot$.
Panels from top to bottom correspond to models with SFE = 0.15, 0.17 and 0.25 respectively.
Colors show different random realizations.
Slightly transparent points correspond to the ending point of the possible magnetar activity at the NS age of 10^5yr, while solid ones correspond to the starting point at age of 10^3yr

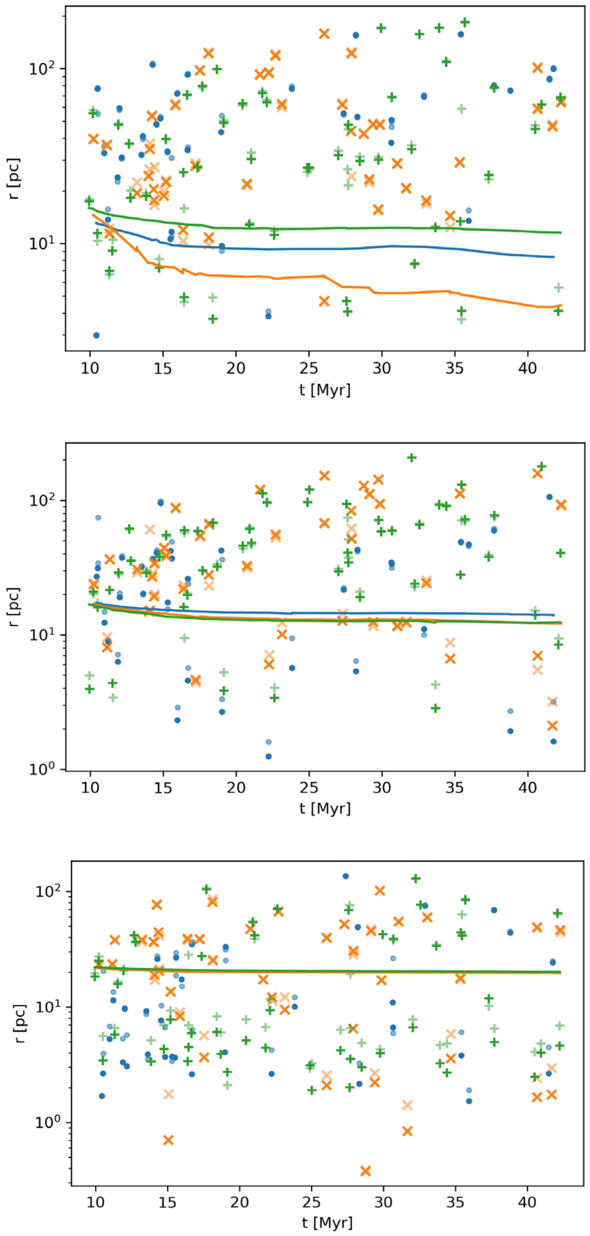

Figure 2 – Distribution of possible FRB candidates around clusters with masses of $M_* = 6000M_\odot$.
Panels from top to bottom correspond to models with SFE = 0.15, 0.17 and 0.25 respectively.
Colors show different random realizations.
Slightly transparent points correspond to the ending point of the possible magnetar activity at the NS age of 10^5yr, while solid ones correspond to the starting point at age of 10^3yr

Figure 2 shows the same picture for clusters with masses of $M_* = 6000M_\odot$. As we can see there is no preference in time for FRBs happening inside or outside of clusters in all models. Indeed, in the case of clusters with the lowest SFE, most of the possible FRB candidates are situated off the cluster, since these are the most expanding clusters surviving with the lowest bound fractions. Nevertheless, this does not prevent FRB candidates located within small clusters at their latest possible times (see yellow and green points below Jacobi radius at the time about 35-40 Myr in the upper panels of Fig 1 and Fig 2).

Since the magnetar life does not exceed 10^5 yrs, the high kick velocity of the order of a few hundred km/s still allows possible FRB candidates to remain in the parent cluster if the progenitor SNe happened deep enough inside the cluster. Especially clusters formed with high SFE can contain about 50-60 percent of all possible FRB candidates within the cluster.

We present the cumulative distribution of distances of all possible magnetars from the cluster edge (i.e., Jacobi radius) in Figure 3. When the distance values are negative, that means the possible FRB source candidate is located inside the cluster. Upper and lower panels correspond to clusters with low and high initial masses, while panels from left to right correspond to different SFEs (SFE=0.15, 0.17, 0.25).

The maximum distance of young neutron stars from their parent clusters as possible FRB progenitors. Analyzing our 18 simulations we have found that the furthest possible FRB source has traveled up to 206 pc from the parent cluster. This happened in one of the model clusters with $M_* = 6000M_\odot$, SFE=0.17. This fact shows that the lowest SFE (i.e., the most post-gas-expulsion expansion) does not mean the furthest kicked NS. But, the lowest SFE kicks out the most neutron stars from the parent cluster. As we can see from Fig 3, the most possible FRB source candidates remain within 100 pc distance from the parent clusters and only a few exceed the distance of 150 pc. But none of the FRB candidate sources reached as far as 250 pc distance from their parent clusters.

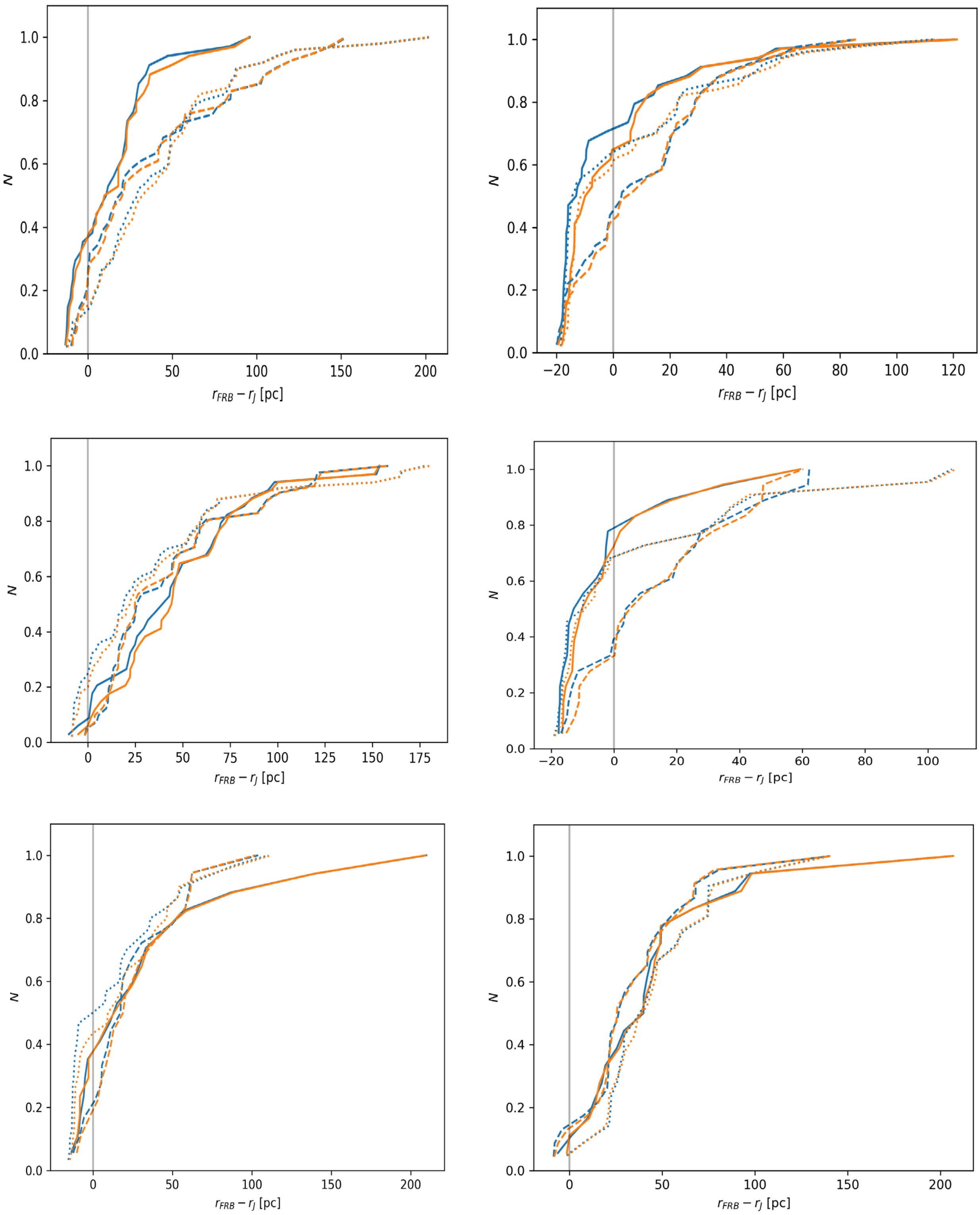

Figure 3 – Cumulative spatial distribution profiles of all possible FRB candidates around clusters with masses of $M_* = 3000M_\odot$ in upper panels and of $M_* = 6000M_\odot$ in lower panels. Distances of possible FRB candidates are corrected for Jacobi radius at a corresponding time. That is, the X-axis shows the distance of neutron stars from the Jacobi radius, where negative values correspond to in-cluster objects. Panels from left to right correspond to models with SFE = 0.15, 0.17 and 0.25 respectively. Line styles show different random realizations. Blue lines and orange lines correspond to the beginning and ending of possible magnetar phases corresponding to NS ages of 10^3yr and 10^5yr, respectively

Conclusion

We have performed direct N-body simulations of the formation and early evolution of star cluster models with different masses and star-formation efficiencies. We have traced all neutron stars – remnants of core-collapse SNe in our simulations during their age range of 10^3-10^5 years, which is believed to be neutron star ages when magnetar activity is possible. Therefore, we consider these neutron stars as possible FRB candidates.

We have shown that FRB sources can be in the clusters as well as around their parent cluster. The fraction of possible FRB sources remaining within the parent cluster increases with the star-formation efficiency but does not correlate with the cluster mass. There is no preference for time for FRB events to happen within or outside of the parent cluster. We have found that the majority of possible FRB sources remain within about 100 pc distance from the parent cluster and only a few candidates can escape the cluster farther than 150 pc. We did not find any FRB candidate farther than 206 pc from the parent cluster in our simulations. This means that almost all FRB source candidates remain within the dissolving parent giant molecular cloud, which contains a lot of ionized gas expelled from the recent star-forming region, therefore might contribute to the dispersion measures of FRBs. But this needs to be tested with high-precision hydrodynamical simulations.

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