THE OCP RELAXATION TIME: THE MOMENT METHOD VS. THE CORRECTED RPA

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An expression is obtained for the visco-elastic relaxation time in one-component plasmas (OCP) from the comparison of the moments-generated dynamic structure factor and that in the local-field corrected random-phase approximation (RPA).

In a classical OCP all physical characteristics are parameterized by the unique dimensionless parameter Γ related to the inverse temperature β, number density of ions *n* and the Wigner-Seitz radius *r* by a simple relation [1]

$$4\pi e^2 \beta n a^2 = 3\Gamma .$$

It also follows from the fluctuation-dissipation theorem (FDT) that the static structure factor *S(k)* is directly related to the static local-field correction *G(k)* [1]:

$$S(k) = \frac{1}{1+\beta n\phi(k)\left[1-G(k)\right]} = \left\{1+\frac{3\Gamma}{q^2}\left[1-G(k)\right]\right\}^{-1},$$

where $q = ka$ and $\phi(k) = 4\pi e^2 / k^2$.

By definition, within the moment approach [2], the visco-elastic relaxation time τ [3] is determined, in terms of the Nevanlinna parameter function as *i/Q* (*q,0*).

As it was shown in [4], the zero-frequency value of the Nevanlinna function is related, via the FDT,

$$\tilde{S}(q,\omega) = \frac{S(k,\omega)}{n} = \frac{1}{\pi\beta n\phi(k)}\left(-\frac{\mathrm{Im}\,\varepsilon^{-1}(k,\omega)}{\omega}\right) = \frac{q^2}{3\pi\Gamma} L(q,\omega)$$

On the other hand, the method of moments gives, for the OCP dynamic structure factor the following expression:

$$S(q,\omega) = \frac{S(q)}{\pi}\,\frac{\omega_1^2\tau^{-1}(\omega_2^2-\omega_1^2)}{\omega^2(\omega^2-\omega_2^2)^2+\tau^{-2}(\omega^2-\omega_1^2)^2}, \qquad (1)$$

where

$$\omega_2^2 = \omega_2^2(q) = \frac{S_4(q)}{S_2(q)} = \omega_p^2\left[1+\frac{q^2}{\Gamma}+I(q)\right];$$

$$\omega_1^2 = \omega_1^2(q) = \frac{S_2(q)}{S_0(q)} = \omega_p^2\left[1-\varepsilon^{-1}(q,0)\right],$$

the frequency moments of the structure factor are directly related to the moments of the loss function:

$$S_\nu(k)=\frac{1}{n}\int_{-\infty}^{\infty}\omega^\nu S(k,\omega)d\omega=\frac{q^2}{3\pi\Gamma}\int_{-\infty}^{\infty}\omega^\nu L(k,\omega)d\omega=\frac{q^2}{3\Gamma}C_\nu(q),$$

and in a OCP we have:

$$S_0(q)=S(q)=\frac{q^2}{3\Gamma}\left[1-\varepsilon^{-1}(q,0)\right],$$

$$S_2(q)=\frac{q^2}{3\Gamma}\omega_p^2,$$

$$S_4(q)=\frac{\omega_p^4q^2}{3\Gamma}\left[1+\frac{q^2}{\Gamma}+I(q)\right].$$

Hence, the relaxation time is determined by the static value of the dynamic structure factor in (1):

$$\tau=\frac{\pi\tilde{S}(q,0)}{S(q)\left(\frac{\omega_2^2}{\omega_1^2}-1\right)}=\frac{\pi\tilde{S}(q,0)}{S(q)C_0(q)\Delta(q)}=\frac{q^2}{3\Gamma}\frac{\pi\tilde{S}(q,0)}{\left[S(q)\right]^2\Delta(q)},\qquad(2)$$

so that $\Delta(q)=(\omega_2^2-\omega_1^2)/\omega_p^2$.

The static characteristics: *S(q),* and the zero-frequency value of the dynamic factor, can be calculated, for example, in the hyper-netted approximation [5].

Alternatively, the dynamic local-field correction $G(\kappa,\omega)$ measures the deviation of the dielectric function from the RPA:

$$\varepsilon(k,\omega)=1+\frac{\phi(k)\Pi(k,\omega)}{1-\phi(k)\Pi(k,\omega)G(k,\omega)},\qquad(3)$$

where $\Pi(\kappa,\omega)$ is the RPA polarization function. For a classical OCP

$$\Pi(k,\omega)=\beta n\left[1+uZ(u)\right],$$

$$u=\frac{\omega}{k}\sqrt{\frac{\beta m}{2}},$$

$$Z(u)=\lim_{\eta\downarrow0}\frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}\frac{e^{-t^2}dt}{t-u-i\eta},$$

so that

$$\frac{\mathrm{Im}\,\phi(k)\Pi(k,\omega)}{\omega}=\sqrt{\pi}\beta nue^{-u^2}\phi(k)=\frac{3\Gamma a}{q^3}\sqrt{\frac{\pi\beta m}{2}}e^{-u^2}.\qquad(4)$$

The relaxation time can be determined by comparing the moment and the local-field corrected RPA expressions for the static value of the loss function or the dynamic structure factor.

Observe that expression (2) follows also from the definition of the zero moment $C_0(k) = 1 - \varepsilon^{-1}(k, \omega = 0)$ in terms of the static local-field correction. Notice that in a classical system $\Pi(\kappa, \omega = 0) = \beta n$ and

$$C_0(k) = 1 - \varepsilon^{-1}(k, \omega = 0) = \frac{3\Gamma}{q^2} S(q)$$

Thus

$$L(k, \omega = 0) = \lim_{\omega \to 0} \left(-\frac{1}{\omega} \operatorname{Im} \frac{1}{1 + \dfrac{\phi(k)\Pi(k, \omega)}{1 - \phi(k)\Pi(k, \omega)G(k, \omega)}} \right) =$$

$$\frac{3\Gamma a}{q^3} \sqrt{\frac{\pi\beta m}{2}} [S(k)]^2$$

Finally,

$$\tilde{S}(q, 0) = \frac{\sqrt{3\Gamma / 2\pi}}{\omega_p q} \left\{ 1 + \frac{3\Gamma}{q^2} \left[1 - G(q) \right] \right\}^{-2} = \frac{\sqrt{3\Gamma / 2\pi}}{\omega_p q} \left[S(q) \right]^2$$

wherefrom

$$\tau\omega_p = \sqrt{\frac{\pi}{6\Gamma}} \frac{q}{\Delta(q)} .$$

The q - dependence of the relaxation time will significantly influence the dispersion relation and determine the decay of the plasmon mode in the OCP.

References

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ВРЕМЯ РЕЛАКСАЦИИ ДЛЯ ОДНОКОМПОНЕНТНОЙ ПЛАЗМЫ: МЕТОД МОМЕНТОВ

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В рамках вязко-упругого приближения найдено выражение для времени релаксации, которое получено из сравнения динамических структурных факторов, рассчитанных как методом моментов, так и для приближения случайных фаз с учетом поправок на локальные поля.

БІР КОМПОНЕНТТІ ПЛАЗМА ҮШІН РЕЛАКСАЦИЯ УАҚЫТЫ: МОМЕНТТЕР ӘДІСІ

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Тұтқырлы серпімді жуықтау шеңберінде релаксация уақыты үшін динамикалық құрылымдық факторларды салыстыру арқылы алынған теңдеу табылды. Бұл факторлар моменттер әдісімен де, және де кездейсоқ фазаларды жергілікті өрістерді қарастырылған жағдайларда есептелді.