INVESTIGATION OF DIELECTRIC PROPERTIES OF NONIDEAL PLASMA IN THE FRAMEWORK OF PSEUDOPOTENTIAL MODEL

V.V. Voronkov
Kazakh National University named after al-Farabi

Spectrum and damping decrements of Langmuir and ionsound waves in nonideal plasma are found. Potential considering quantum effects of diffraction is used. Local fields are considered through the theory of linear dielectric response and numerical solution of Ornstein-Zernicke equation in hypernetted approach.

1 Pseudopotential model

For proper investigation of dielectric properties of dense plasma one should consider quantum effects and local fields, playing significant role in such plasma [1]. For that aim pseudopotential, considering quantum effects of diffraction and local fields is constructed in the framework of linear dielectric response theory.

The effective potential considering quantum effects was proposed by Deutch and co-authors in work [2]:

$$\varphi_{ab}(r) = \frac{e_a e_b}{r}\left[1 - \exp\left(-\frac{r}{\lambda_{ab}}\right)\right], \tag{1}$$

where $\lambda_{ab} = \hbar/\left(2\pi\mu_{ab}k_B T\right)^{1/2}$ - refers to the thermal de Broglie wavelength, $\mu_{\alpha\beta}$ is the reduced mass of interacting particles, $T_{\alpha\beta} = \left(m_\alpha T_\beta + m_\beta T_\alpha\right)/\left(m_\alpha + m_\beta\right)$. In expression (1) the exponential term with the thermal de Broglie wavelength accounts for the quantum effects of diffraction. It is worthwhile to notice that if expressed in plasma parameters, the thermal de Broglie wavelength is $\lambda_{\alpha\beta}/a \sim (\Gamma/r_s)^{1/2}$ (a is the average interionic distance $a=[3/(4\pi n)]^{1/3}$, n_i is the ion number density, $\Gamma=e^2/(ak_BT)$ is the coupling parameter, $r_s=am_e e^2/\hbar^2$ is the dimensionless density parameter) thus, the less value of the parameter r_s is chosen, the more influential quantum effects upon potential (2) are, that is the more dense plasma, the more significant quantum effects are. One can see that considering quantum effects in potential (1) leads to decrease in forces, acting between particles (decrease in electron-electron and ion-ion repulsion and electron-ion attraction), that is plasma became less "elastic".

In order to consider local fields one can use local field functions. Electron-electron local field function $G_{ee}(k)$ can be found by the following formula:

$$G_{ee}(k) = 1 + k_B T\,\frac{\tilde{C}_e(k)}{\tilde{\varphi}_{ee}(k)}. \tag{2}$$

Here C_e is the electronic direct correlation function, obtained from the solution of HNC equation, with the aid of scheme proposed in [3], using Fourier transform of electron-electron potential $\tilde{\varphi}_{ee}(k)$ (1).

The ion-ion pseudopotential, taking into account local field correction, quantum effects and the electron screening of ions is derived in the framework of the linear density-response formalism:

$$\tilde{\Phi}_{ii}(k) = \tilde{\varphi}_{ii}(k)\left(1 - \frac{\tilde{\varphi}_{ee}(k)}{\dfrac{k_B T}{n_e} + \tilde{\varphi}_{ee}(k)\left(1 - G_{ee}(k)\right)}\right). \tag{3}$$

Unlike the Debye and the Coulomb potentials (which are also screened), pseudopotential (3) is finite at $r=0$, because it takes into account quantum effects, like the Deutch potential (1). For small values of the coupling parameter $\Gamma<1$ and at small interparticle distances, ion-ion pseudopotential (3) approches to the ion-ion Deutch potential (1), as both potentials consider quantum effects of diffraction, and at large interparticle distances approaches to the Debye potential respectively, as they both considers the screening phenomena. If one puts Fourier images of the Colomb potential into expression (3), $\varphi_{ee}=\varphi_{ii}=4\pi\Gamma/k^2$, and takes local fields equal zero, $\tilde{G}_{ee}(k)=0$, then one gets the Fourier image of the Debye-Huckel potential $\Phi=4\pi\Gamma/\left(k^2+1/r_D{}^2\right)$, what demonstrates us screening properties of pseudopotential (3). At large values of Γ, pseudopotential (3) and the Debye-Huckel potential start to diverge as the more Γ, the more influential local fields are.

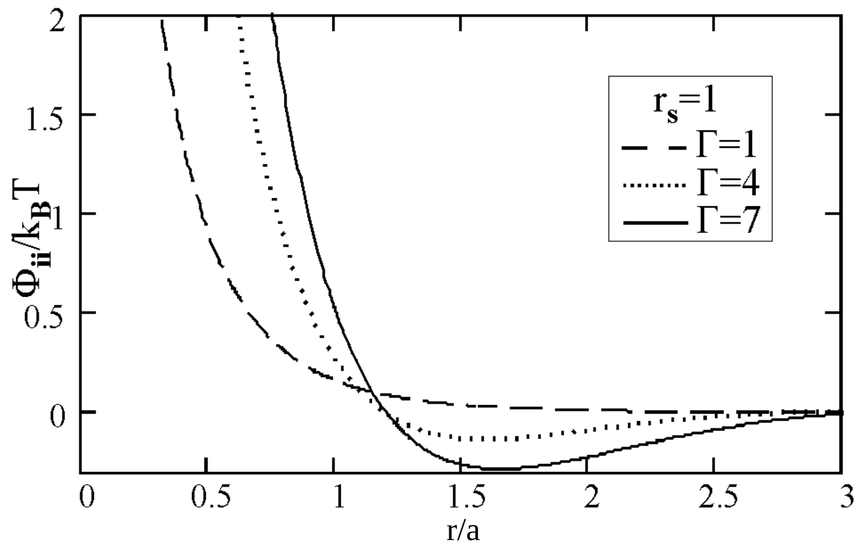

Fig.1: Ion-ion pseudopotential (3) at $r_s=1$ for various Γ

As Fig.1 demonstrates, when Γ grows the ion-ion pseudopotential acquires potential well, deepening with increase in Γ. As numerical calculations show, the appearance of the potential well is caused by the local fields. The depth of the well increases with the decrease in r_s, and, therefore, the quantum effects deepen the well.

Having found the ion-ion pseudopotential, one can calculate ion-ion local field function via the following expressions:

$$G_{ii}(k)=\frac{1}{\varepsilon_e(k)}+k_BT\frac{\tilde{C}_i(k)}{\tilde{\varphi}_{ii}(k)}, \tag{4}$$

$$\varepsilon_e(k)=1+\frac{\tilde{\varphi}_{ee}(k)}{\frac{k_BT}{n_e}-\tilde{\varphi}_{ee}(k)G_{ee}(k)}, \tag{5}$$

$\varepsilon_e(k)$ is the electronic dielectric function, $\tilde{C}_i(k)$ symbolizes the Fourier transform of ionic direct correlation function, obtained from the numerical solution of the Ornstein-Zernike equation in the hypernetted approximation for pseudopotential (3). In this case the Ornstein-Zernike equation is solved without the aid of an auxiliary function as in [3], as pseudopotential (3) is screened, unlike potential (1), which is close to the Colomb potential at large distances.

Hereby using effective potential (1), in the framework of linear dielectric function we constructed the pseudopotential model, considering quantum effects of diffraction, local fields and screening effects, playing significant role in dense plasma. With the aid of this pseudopotential model we further investigate the dielectric properties of nonideal plasma.

2. High frequency waves propagation in an nonideal hydrogen plasma.

Theory of linear dielectric function says that the dielectric function of a plasma can be written as:

$$\frac{1}{\varepsilon(k,\omega)} = 1 + \sum_{q,k} \tilde{\varphi}_{qk}(k)\chi_{qk}(k,\omega) , \tag{6}$$

$$\begin{aligned}
\chi_{ee} &= \chi_e^{(0)} \frac{1-\chi_i\tilde{\varphi}_{ii}\left(1-\tilde{G}_{ii}\right)}{D}, \\
\chi_{ii} &= \chi_i^{(0)} \frac{1-\chi_e\tilde{\varphi}_{ee}\left(1-\tilde{G}_{ee}\right)}{D}, \\
\chi_{ie} &= \chi_e^{(0)}\chi_i^{(0)} \frac{\tilde{\varphi}_{ie}}{D}, \\
\chi_{ei} &= \chi_i^{(0)}\chi_e^{(0)} \frac{\tilde{\varphi}_{ei}}{D},
\end{aligned} \tag{7}$$

$$D = \left(1-\chi_e^{(0)}\tilde{\varphi}_{ee}\left(1-\tilde{G}_{ee}\right)\right)\left(1-\chi_i^{(0)}\tilde{\varphi}_{ii}\left(1-\tilde{G}_{ii}\right)\right) - \chi_e^{(0)}\chi_i^{(0)}\tilde{\varphi}_{ei}\tilde{\varphi}_{ie}, \tag{8}$$

here $\chi_{ab}(\mathbf{k},\omega)$ is the response function of particles of kinds *a* and *b*, $\tilde{\varphi}_{ab}(k)$ - Fourier transform of potential of interaction of particles, $\tilde{G}_{ab}(k)$ - local field functions, the screened functions $\chi_a^{(0)}(k,\omega)$can be found via the following formula:

$$\chi_a^{(0)}(\mathbf{k},\omega) = -\frac{n_a}{k_B T} W\left(\frac{\omega}{k\upsilon_{T_a}}\right). \tag{9}$$

Here $\upsilon_{T_a} = \left(k_B T/m_a\right)^{1/2}$ is the thermal velocity of particles *a*, function $W(z)$ is a well known function:

$$W(z) = 1 - z\exp\left(-z^2/2\right)\int_0^z \exp\left(y^2/2\right)dy + i\sqrt{\frac{\pi}{2}} z\exp\left(-z^2/2\right), \tag{10}$$

having asymptotical expansion at z<1:

$$W(z) = i\sqrt{\frac{\pi}{2}} z\exp\left(-\frac{z^2}{2}\right) + 1 - z^2 + \frac{z^4}{3} - ... + \frac{(-1)^{n+1}}{(2n+1)!!} z^{2n+2} \tag{11}$$

and at z>1:

$$W(z)=i\sqrt{\frac{\pi}{2}}z\exp\left(-\frac{z^2}{2}\right)-\frac{1}{z^2}-\frac{3}{z^4}-...-\frac{(2n-1)!!}{z^{2n}}\,. \qquad (12)$$

Putting expressions (7)-(8) into (6), we get the expression for dielectric function:

$$\varepsilon^l=\frac{D}{\left[\begin{array}{l}(1+\varphi_{ee})\left(\chi_e^{(0)}-\chi_e^{(0)}\chi_i^{(0)}\varphi_{ii}(1-G_{ii})\right)+\varphi_{ii}\chi_i^{(0)}-\chi_e^{(0)}\chi_i^{(0)}\varphi_{ee}\varphi_{ii}(1-G_{ee})+\\+2\chi_e^{(0)}\chi_i^{(0)}\varphi_{ei}\varphi_{ie}\end{array}\right]}. \qquad (13)$$

It is well known [4], that the condition of existence of longitudinal waves is

$$\varepsilon^l(k,\omega)=0\,, \qquad (14)$$

thus one gets

$$\left(1-\chi_e^{(0)}\varphi_{ee}[1-G_{ee}]\right)\left(1-\chi_i^{(0)}\varphi_{ii}[1-G_{ii}]\right)-\chi_e^{(0)}\chi_i^{(0)}\varphi_{ei}\varphi_{ie}[1-G_{ei}][1-G_{ie}]=0, \qquad (15)$$

where $\tilde{\varphi}_{qk}(k,\omega)=\dfrac{4\pi e^2}{k^2\left(1+k^2\lambda_{qk}^2\right)}$ - Fourier transform of potential (1).

Having solved equation (15), using (9), the dispersion relation is obtained:

$$\begin{aligned}&1+\frac{\omega_{Le}^2(1-G_{ee})}{k^2V_{Te}^2\left(1+k^2\lambda_{ee}^2\right)}W\left(\frac{\omega}{kV_{Te}}\right)+\frac{\omega_{Li}^2(1-G_{ii})}{k^2V_{Ti}^2\left(1+k^2\lambda_{ii}^2\right)}W\left(\frac{\omega}{kV_{Ti}}\right)-\\&-\frac{\omega_{Le}^2(1-G_{ee})}{k^2V_{Te}^2\left(1+k^2\lambda_{ee}^2\right)}W\left(\frac{\omega}{kV_{Te}}\right)\cdot\frac{\omega_{Li}^2(1-G_{ii})}{k^2V_{Ti}^2\left(1+k^2\lambda_{ii}^2\right)}W\left(\frac{\omega}{kV_{Ti}}\right)+\\&+\frac{\omega_{Le}^2(1-G_{ei})}{k^2V_{Te}^2\left(1+k^2\lambda_{ei}^2\right)}W\left(\frac{\omega}{kV_{Te}}\right)\cdot\frac{\omega_{Li}^2(1-G_{ie})}{k^2V_{Ti}^2\left(1+k^2\lambda_{ie}^2\right)}W\left(\frac{\omega}{kV_{Ti}}\right)=0.\end{aligned} \qquad (16)$$

The phase velocity of high frequency waves is much greater then the thermal velocities of charged particles $\frac{\omega}{k}\gg V_{Te},V_{Ti}$, thus using formula (12) one can get the expression for the dispersion relation of plasma:

$$1+i\sqrt{\frac{\pi}{2}}\frac{\omega\omega_{Le}^2(1-G_{ee})}{k^3V_{Te}^3\left(1+k^2\lambda_{ee}^2\right)}\exp\left(-\frac{\omega^2}{2k^2V_{Te}^2}\right)-\frac{\omega_{Le}^2}{\omega^2}\left(1+3\frac{k^2V_{Te}^2}{\omega^2}\right)\frac{(1-G_{ee})}{\left(1+k^2\lambda_{ee}^2\right)}=0\,. \qquad (17)$$

Using the condition of existence of propagation of longitudinal waves in plasma (14) and expression for damping decrement [4]

$$\delta(k) = -\frac{\operatorname{Im}\varepsilon(\omega,k)}{\dfrac{\partial \operatorname{Re}\varepsilon(\omega,k)}{\partial\omega}}, \tag{18}$$

one can obtain spectra *ω(k)* and damping decrement *δ(k)* of plasma waves:

$$\omega = \omega_{Le}\left(1+\frac{3}{2}k^2 r_{De}^2\right)\sqrt{\frac{1-G_{ee}}{1+k^2\lambda_{ee}^2}}, \tag{19}$$

$$\delta = -\sqrt{\frac{\pi}{8}}\cdot\frac{\omega_{Le}(1-G_{ee})}{k^3 r_{De}^3(1+k^2\lambda_{ee}^2)}e^{-\left(\frac{3}{2}+\frac{1}{2k^2 r_{De}^2}\right)\frac{1-G_{ee}}{1+k^2\lambda_{ee}^2}}. \tag{20}$$

One can see from formula (19), that considering quantum-mechanical effects of diffraction of electrons leads to decrease in Langmuir waves frequency, what can be explained that, as it was aforesaid, electron diffraction decreases force of repulsion between electrons and the electron gas becomes less “elastic”. Local field corrections diminishes the Langmuir frequency as the electron-electron functions of local fields are positive. Formula (19) shows that when plasma density grows, the dispersion of Langmuir waves can turn to anomalous $\omega(k) < \omega_{Le}$. Such dispersion of plasmons, shown in fig. 2 was predicted in work [1] on the basis of pseudopotential model, considering quantum effects and local fields and in work [5] on the basis of molecular dynamics data. Collisionless damping decrement (20) is changed in the result of change of phase velocity of plasma waves $V_{phase}=\omega/k$ due to change in spectra. Formulae (19), (20) are generalizations of corresponding formulae for classical plasma [4].

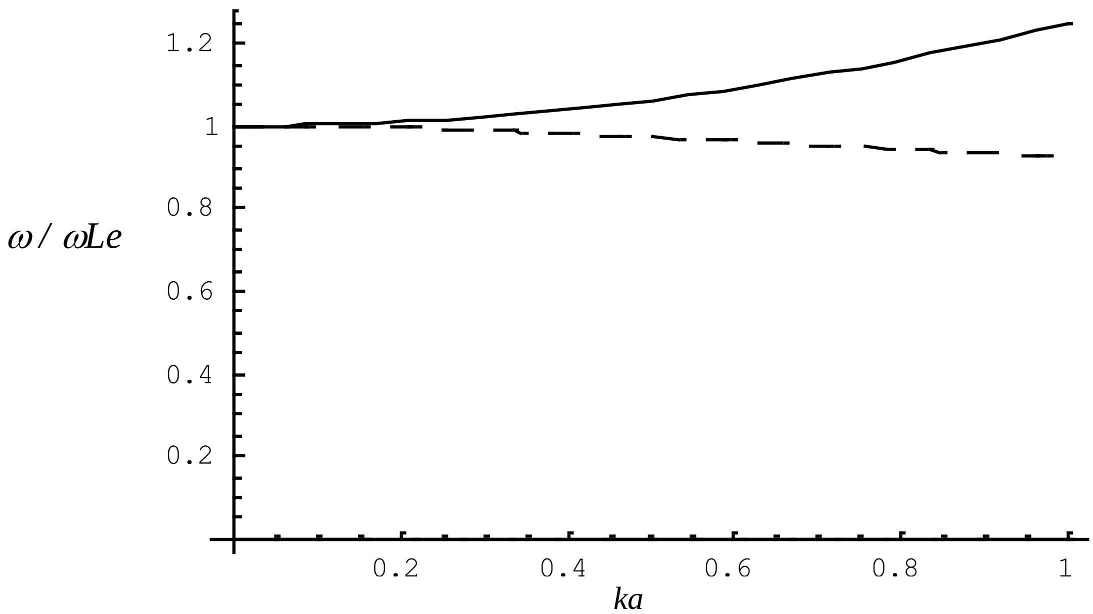

Fig 2. Langmuir spectrum at *Γ=2, rs=1*.
Solid line – classical plasma, dash line – formula (19).

2. Low frequency waves propagation in an nonideal hydrogen plasma.

The condition of the low frequency mode is $V_{Ti} \ll \frac{\omega}{k} \ll V_{Te}$, using formulae (11), (12), (16) the dispersion relation can be written as:

$$
\begin{aligned}
&1+\frac{\omega_{Le}^2(1-G_{ee})}{k^2V_{Te}^2(1+k^2\lambda_{ee}^2)}\left(1+i\sqrt{\frac{\pi}{2}}\frac{\omega}{k\ V_{Te}}\right)-\frac{\omega_{Le}^2\omega_{Li}^2\omega(1-G_{ee})(1-G_{ii})}{k^5V_{Te}^2V_{Ti}^3(1+k^2\lambda_{ee}^2)(1+k^2\lambda_{ii}^2)}\times\\
&\left(1+i\sqrt{\frac{\pi}{2}}\frac{\omega}{k\ V_{Te}}\right)\exp\left(-\frac{\omega^2}{2k^2V_{Ti}^2}\right)+\frac{\omega_{Le}^2\omega_{Li}^2\omega(1-G_{ei})^2}{k^5V_{Te}^2V_{Ti}^3(1+k^2\lambda_{ei}^2)^2}\left(1+i\sqrt{\frac{\pi}{2}}\frac{\omega}{k\ V_{Te}}\right)\times\\
&\times\exp\left(-\frac{\omega^2}{2k^2V_{Ti}^2}\right)+\frac{\omega_{Li}^2\omega(1-G_{ii})}{k^3V_{Ti}^3(1+k^2\lambda_{ii}^2)}i\sqrt{\frac{\pi}{2}}\exp\left(-\frac{\omega^2}{2k^2V_{Ti}^2}\right)-\frac{\omega_{Li}^2(1-G_{ii})}{\omega^2(1+k^2\lambda_{ii}^2)}+\\
&+\frac{\omega_{Le}^2\omega_{Li}^2(1-G_{ee})(1-G_{ii})}{\omega^2k^2V_{Te}^2(1+k^2\lambda_{ee}^2)(1+k^2\lambda_{ii}^2)}-\frac{\omega_{Le}^2\omega_{Li}^2(1-G_{ei})(1-G_{ie})}{\omega^2k^2V_{Te}^2(1+k^2\lambda_{ei}^2)(1+k^2\lambda_{ie}^2)}=0
\end{aligned}
\tag{21}
$$

Using formulae (14), (18), one can get spectrum *ω(k)* and damping decrement *δ(k)* of low frequency longitudinal waves:

$$
\omega=\omega_{Li}\sqrt{\frac{\dfrac{1-G_{ii}}{1+k^2\lambda_{ii}^2}+\dfrac{B}{k^2r_{De}^2}}{1+\dfrac{1-G_{ee}}{k^2r_{De}^2(1+k^2\lambda_{ee}^2)}}},
\tag{22}
$$

$$
\delta=-\sqrt{\frac{\pi}{8}}\frac{\omega^4}{k^3}\left(\frac{1-G_{ii}}{1+k^2\lambda_{ii}^2}+\frac{A}{k^2r_{De}^2}\right)\left[\begin{aligned}&\frac{1}{V_{Ti}^3}\left(\frac{\omega_{Le}^2\omega B}{k^3V_{Te}^3}+\frac{1-G_{ii}}{1+k^2\lambda_{ii}^2}\right)\exp\left(-\frac{\omega^2}{2k^2V_{Ti}^2}\right)+\\&+Z\frac{M}{m}\frac{1-G_{ee}}{V_{Te}^3(1+k^2\lambda_{ee}^2)}\end{aligned}\right],
\tag{23}
$$

where $A=\frac{1}{(1+k^2\lambda_{ee}^2)(1+k^2\lambda_{ii}^2)}-\frac{1}{(1+k^2\lambda_{ei}^2)^2}$, $B=\frac{1}{(1+k^2\lambda_{ei}^2)^2}-\frac{(1-G_{ee})(1-G_{ii})}{(1+k^2\lambda_{ee}^2)(1+k^2\lambda_{ii}^2)}$.

It is easy to notice, that formulae (22) and (23) are generalizations of classical formulae for low-density plasma [4].

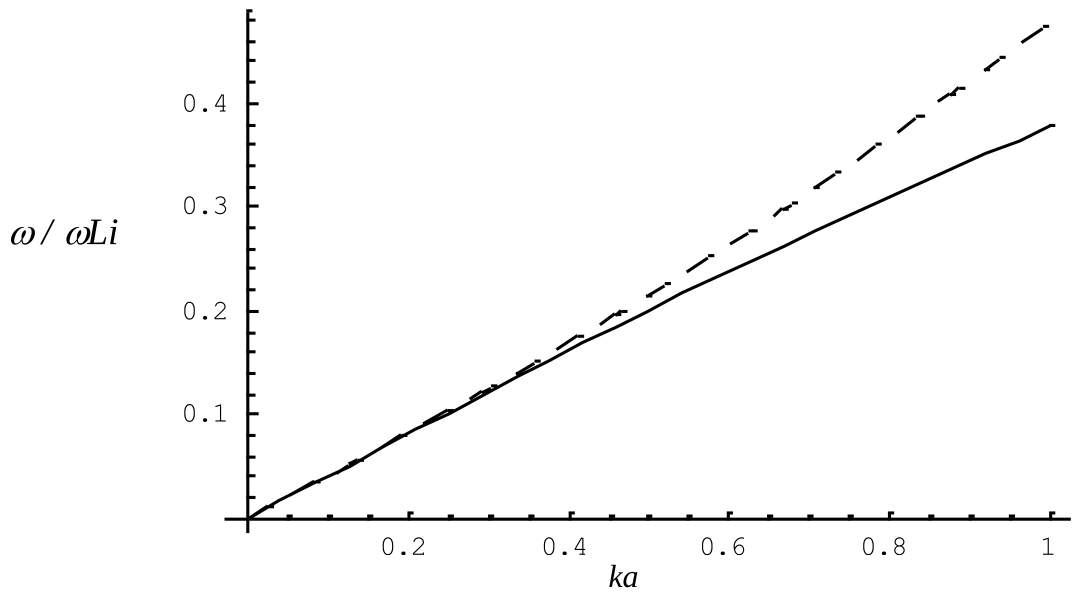

Fig.3. Ionsound waves spectra at *Γ=2, rs=1*.
Solid line – classical plasma, dash line – formula (22).

Formula (22) shows that quantum-mechanical effects of electron diffraction increase, and those of ions decrease ionsound wave frequency. Frequency of oscillations in a dense plasma is higher then classical plasma oscillations, because the electron thermal de Broglie wavelength is considerably greater then the ion thermal de Broglie wavelength, and contribution of the second term in the numerator of the subduplicate fraction is insignificant, because $B \ll 1$. Formula (22) also shows that influence of electron-electron local fields increases and ion-ion local fields diminishes ionsound frequency. Numerical calculations for various plasma parameters shows that ionsound frequency in dense plasma is higher than that in classical case, wherein the difference between such frequencies increases along with growth of plasma density, what is connected with growth of effects of diffraction and nonideality. The form of damping decrement (2) is also qualitatively changed. Numerical calculations for various plasma parameters says that dense plasma damping decrement is higher then the classical one, wherein the denser plasma the higher difference between both decrements, what is connected with growth of diffraction and nonideality effects.

Conclusion

This work is devoted for investigation of dielectric properties of dense high-temperature plasma. For that aim a pseudopotential model, considering quantum effects of diffraction, local fields and electron screening playing important role in nonideal plasma, is constructed. The pseudopotential model is constructed on the basis of effective potential in the framework of theory of linear dielectric response functions with the use of Ornstein-Zernike integral equation in hypernetted chain approximation. Spectra and damping decrements of Langmuir and ionsound oscillations are obtained with the aid of this pseudopotential model. It is shown that the expressions for spectra and damping decrements are generalizations of classical expressions for rarified plasma. The obtained results develop the theory of linear dielectric response functions and enable a researcher to conduct further systematic investigations of dielectric, thermodynamic and transport properties of nonideal plasma.

References

1. Архипов Ю.В., Баимбетов Ф.Б., Давлетов А.Е., Стариков К.В. Псевдопотенциальная теория плотной высокотемпературной плазмы. – Алматы, 2002, 113 с.
2. Minoo H., Gombert M.M., Deutch C.// Phys. Rev. – 1981. – A 23. – P. 924.

3. Ng K.-C. Hypernetted chain solutions for the classical one-component plasma up to Г=7000 // J.Chem.Phys. - 1974. - Vol.61, №7. - P.2680-2689.
4. Александров А.Ф., Богданкевич Л.С., Рухадзе А.А. Основы электродинамики плазмы. – Москва: Высшая школа, 1988. - 189 с.
5. Hansen J.P., McDonald I.R. Microscopic simulation of a strongly coupled hydrogen plasma // Phys.Rev.A - 1981. - Vol.23, №4. - P.2041-2059.

ПСЕВДОПОТЕНЦИАЛДЫҚ МОДЕЛЬ НЕГІЗІНДЕ ИДЕАЛ ЕМЕС ПЛАЗМАНЫҢ ДИЭЛЕКТРЛІК ҚАСИЕТТЕРІН ЗЕРТТЕУ

В.В. Воронков

Осы макалада псевдопотенциалдық модель негізінде екі компонентті толық иондалған тығыз жоғарғы температурадағы плазманың диэлектрлік қасиеттері зерттелді. Электрондармен экрандалуды, дифракция және симметрия кванттық эффектілерді, электрон-электронды локалды өрістерді ескеретін ион-ионды псевдопотенциал алынған. Локалды өрістердің электрон-электронды функцияларын есептеу үшін Орнштейн-Церниктің интегралды теңдеуін гипертізбекті жуықтауда қолданылады. Кванттық эффектілердің және локалды өрістердің дисперсияға және өшу декрементінің ленгмюрлік және ионды-дыбыс плазмондарына әсері байқалды.

ИССЛЕДОВАНИЕ ДИЭЛЕКТРИЧЕСКИХ СВОЙСТВ НЕИДЕАЛЬНОЙ ПЛАЗМЫ В РАМКАХ ПСЕВДОПОТЕНЦИАЛЬНОЙ МОДЕЛИ

В.В. Воронков

В данной работе с помощью псевдопотенциальной модели исследуются диэлектрические свойства двухкомпонентной полностью ионизованной плотной высокотемпературной плазмы. Для создания псевдопотенциальной модели, учитывающей квантовые эффекты, коллективные явления и эффекты неидеальности используется теория линейного диэлектрического отклика. Интегральное уравнение Орнштейна-Цернике в гиперцепном приближении используется для численного расчёта электрон-электронных и ион-ионных функций локальных полей. Изучено влияние квантовых эффектов и локальных полей на дисперсию ленгмюровских колебаний и ионнозвуковых плазмонов.