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APPLICATION OF GEOMETROTHERMODYNAMICS TO THE SYSTEM WITH ZERO SOUND DESCRIBED BY THE METHOD OF HOLOGRAPHIC DUALITY

In the framework of the method of geometrothermodynamics, in present work, we studied the properties of equilibrium manifold of the system with zero-sound predicted by the holographic duality method. The results are invariant under the Legendre transformations, i.e. independent of the choice of thermodynamic potential. For the systems under consideration, the corresponding metrics, determinants of metrics and scalar curvatures are calculated, and their properties are also described. Using the holographic approach, a new type of quantum liquid was discovered. The heat capacity of the liquid obtained in this work at low temperatures depends on the temperature $\sim T^6$. Entropy, which depends on temperature and baryom density, was taken as the thermodynamic potential. 3-dimensional obtained that clearly show at which values of thermodynamic variables scalar curvatures tend to infinity or to zero, which indicates possible phase transitions and possible compensation of interactions by quantum effects, respectively. It is shown that both variants of metrics in this case lead to the same conclusion regarding the location of possible phase transition lines in the considered holographic system with zero sound.

Keywords: geometrothermodynamics, Legendre transformations, metric tensor, scalar curvature, holographic duality, zero sound.

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Голографиялық дуальдік әдіспен сипатталған нөлдік дыбысы бар жүйеге геометриялық термодинамиканы қолдану

Бұл ұсынылған жұмыста термодинамика геометриясы әдісі аясында голографиялық дуальдік әдіспен болжанған нөлдік дыбысы бар жүйенің тепе-теңдік күйдегі алуан түрлілігінің қасиеттері зерттелді. Лежандр түрлендірулеріне қатысты инвариантты нәтижелер алынды, яғни термодинамикалық потенциалды таңдауға тәуелсіз есептелінді. Осы қарастырылып отырған жүйе үшін тиісті метрикалар мен скалярлы қисықтар есептелініп, олардың қасиеттері сипатталды. Голографиялық тәсіл көмегімен біз осы жұмыста кванттық сұйықтықтың жаңа түрін табып және сұйықтықтың жылу сыйымдылығы төмен температурада қарастырдық. Бұл жұмыста төмен температурада алынған сұйықтықтың жылу сыйымдылығы шамамен $\sim T^6$ температураға тәуелді екені көрсетілген. Термодинамикалық потенциал ретінде температура мен бариондық тығыздыққа байланысты энтропия алынды және де олардың 3-өлшемді графиктері тұрғызылды. Алынған графиктер арқылы ондаға термодинамикалық айнымалы скаляр қисықтары шексіздікке немесе нөлге ұмтылатыны көрінеді, бұл дегеніміз мүмкін болатын фазалық ауысуларды және сәйкесінше кванттық әсермен өзара іреккеттесудің мүмкін болатындығын көрсетеді. Бұл жағдайда метриканың екі нұсқасы да нөлдік дыбысы бар қаралған голографиялық жүйеде ықтимал фазалық ауысулар сызықтарының орналасуы туралы бірдей қорытындыға әкелетіндігі көрсетілген.

Түйін сөздер: геометротермодинамика, Лежандр түрлендірулері, метрикалық тензор, скалярлық қисық, голографиялық екі жақтылық, голографиялық екі жақтылық.

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Применение геометротермодинамики к системе с нулевым звуком описанной методом голографических дуальностей

В настоящей работе в рамках метода геометротермодинамики исследованы свойства равновесного многообразия системы с нулевым звуком, предсказанной методом голографических дуальностей. Получены результаты инвариантные относительно преобразований Лежандра, т. е. независимые от выбора термодинамического потенциала. Для рассматриваемой системы рассчитаны соответствующие метрики и скалярные кривизны, а также описаны их свойства. С помощью голографического подхода в работе нами был обнаружен новый тип квантовой жидкости. Показано, что полученная в этой работе теплоемкость жидкости при низких температурах зависит от температуры $\sim T^6$. В качестве термодинамического потенциала бралась энтропия, зависящая от температуры и барионной плотности. Получены 3-мерные графики, на которых хорошо видно, при каких значениях термодинамических переменных скалярные кривизны стремятся к бесконечности или к нулю, что указывает на возможные фазовые переходы и на возможную компенсацию взаимодействий квантовыми эффектами, соответственно. Показано, что оба варианта метрик в данном случае приводят к одному и тому же выводу относительно расположения линий возможных фазовых переходов в рассмотренной голографической системе с нулевым звуком.

Ключевые слова: геометротермодинамика, преобразования Лежандра, метрический тензор, скалярная кривизна, голографические дуальности, нулевой звук.

Introduction

The method of holographic dualities describes quantum systems in the strong coupling mode [1]. Holographic models lead to a number of predictions that are in good agreement with experimental data. Moreover, new types of quantum systems are predicted using the holographic method. For example, in [2], a system was found that at low temperatures has a zero sound more like a Fermi liquid, but this system has a completely different temperature dependence of the heat capacity. Therefore, the task of studying the thermodynamic properties of new quantum systems by the predicted method of holographic dualities becomes urgent.

In this work, the thermodynamic properties of a holographic system with zero sound were investigated. Geometrical thermodynamics was used as a research method [3], and entropy depending on temperature and density of baryons was used as a thermodynamic potential.

Methods

Geometrothermodynamica

In geometrothermodynamics (GTD) proposed by E. Quevedo [3], interactions in thermodynamic systems are described using the curvature of an equilibrium manifold invariant with respect to Legendre transformations. In thermodynamics, too, the physical properties of a system do not depend on the choice of thermodynamic potentials by which this system is described. The transition from one set of thermodynamic potentials to another is carried out using Legendre transformations, and in this sense, thermodynamics is invariant with respect to Legendre transformations. In GTE, for example, as it was shown in [3], an ideal gas whose particles do not interact with each other corresponds to a manifold with zero curvature. In the case of interacting systems with a non-trivial structure of phase transitions, the GTD reproduces the behavior of the system near the points where phase transitions occur. As was shown by the example of Van der Waals gases, Bose-Einstein gases, thermodynamics of various black holes, etc. [4], near phase transitions, the scalar curvature of the corresponding equilibrium manifolds tends to infinity. This fact is convenient for searching for unknown phase transitions in poorly studied thermodynamic systems.

In this paper, to study thermodynamic systems, we calculated metric tensors of the corresponding equilibrium manifolds, determinants of metric tensors and corresponding scalar curvatures. As formulas for calculating metrics and corresponding metric tensors, we used [3]:

$$dl^2 = E_a \frac{\partial \Phi}{\partial E^a} \delta_{ab} \frac{\partial^2 \Phi}{\partial E^b \partial E^c} dE^a E^c, \qquad (1)$$

$$dl^2 = E_a \frac{\partial \Phi}{\partial E^a} \eta_{ab} \frac{\partial^2 \Phi}{\partial E^b \partial E^c} dE^a E^c. \qquad (2)$$

l^2 is the square of the thermodynamic length, $\Phi \equiv \Phi(E^a)$ is the thermodynamic potential, which clearly depends on other thermodynamic potentials - E^a, (a =1, …, n), n is the number of thermodynamic potentials on which it depends Φ, $\delta_{ab} = diag(1,1,...,1)$ и $\eta_{a,b} = diag(1,-1,...,-1)$. Both relations (1) and (2) are invariant with respect to Legendre transformations [3].

The expression for the curvature tensor has the usual form:

$$R_{abcd} = \frac{1}{2}\left(\frac{\partial^2 g_{ad}}{\partial E^b \partial E^c} + \frac{\partial^2 g_{bc}}{\partial E^a \partial E^d} - \frac{\partial^2 g_{ac}}{\partial E^b \partial E^d} - \frac{\partial^2 g_{bd}}{\partial E^a \partial E^c}\right) + g_{np}\left(\Gamma^n_{bc}\Gamma^p_{ad} - \Gamma^n_{bd}\Gamma^p_{ac}\right). \qquad (3)$$

Where $g^{nm}(g_{ad})$ is metric tensor, $\Gamma^n_{bc} = \frac{1}{2} g^{nm}\left(\frac{\partial g_{mb}}{\partial E^c} + \frac{\partial g_{mc}}{\partial E^b} - \frac{\partial g_{bc}}{\partial E^m}\right)$ are Christoffel symbols.

Further, the scalar curvature is calculated by the formula $R = g^{ac} g^{bd} R_{abcd}$. Since in the future we will deal with systems that depend only on two thermodynamic potentials, the expression for scalar curvature is simplified to:

$$R = \frac{2P_{1212}}{\det(g)}, \qquad (4)$$

where $\det(g)$ is the determinant of a two-dimensional metric tensor.

Results and Discussion

A system with zero sound from a holographic description

Using the holographic approach, a new type of quantum liquid was discovered in [2]. The heat capacity of the liquid obtained in this work at low temperatures depends on the temperature $\sim T^6$. Despite the unusual behavior of the heat capacity for Fermi liquids, the system has a zero sound mode at low temperatures. In [2], an expression for the entropy of this liquid is given in the approximation $Td^{-\frac{1}{p}} \prec\prec 1$:

$$S(T,d) = S_0 + N_q \left(\frac{4\pi}{p+1}\right)^{2p+1} \left(\frac{T^{2p}}{2d}\right). \qquad (5)$$

Where T is temperature, d is baryon density, p is dimension of space (we took 3), in which this liquid is considered, S_0 is entropy at zero temperature and N_q is some constant. Figure 1 shows the graph (5) for $\frac{N_q}{2}\left(\frac{4\pi}{p+1}\right)^{2p+1} \equiv 1$ and some range of parameters T and d.

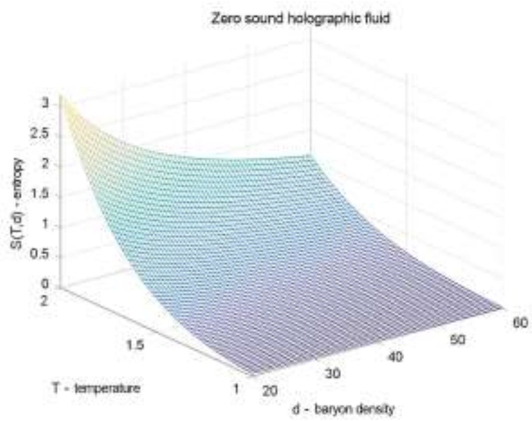


Figure 1. Entropy (5) as a function of temperature and density of baryons for a holographic system with zero sound [2].

Applying to expression (5) the formula for metric (1) and, to simplify $\frac{N_q}{2}\left(\frac{4\pi}{p+1}\right)^{2p+1}$, taking as one, we get the metric tensor:

$$g(T,d) = \begin{bmatrix} \frac{180\, T^{10}}{d^2} & -\frac{15\, T^{11}}{d^3} \\ -\frac{15 T^{11}}{d^3} & -\frac{2 T^{12}}{d^4} \end{bmatrix}. \qquad (6)$$

Next , the determinant of this tensor:

$$\det(g) = -\frac{585\, T^{22}}{d^6}. \qquad (7)$$

And scalar curvature (4):

$$R = -\frac{985\, d^2}{135\, T^{12}}. \qquad (8)$$

Applying the same formula (2) to expression (5) and also taking $\frac{N_q}{2}\left(\frac{4\pi}{p+1}\right)^{2p+1}$ per unit, we get a metric tensor:

$$g(T,d)=\begin{bmatrix} \frac{180\,T^{10}}{d^2} & -\frac{21T^{11}}{d^3} \\ -\frac{21T^{11}}{d^3} & -\frac{2\,T^{12}}{d^4} \end{bmatrix}. \quad (9)$$

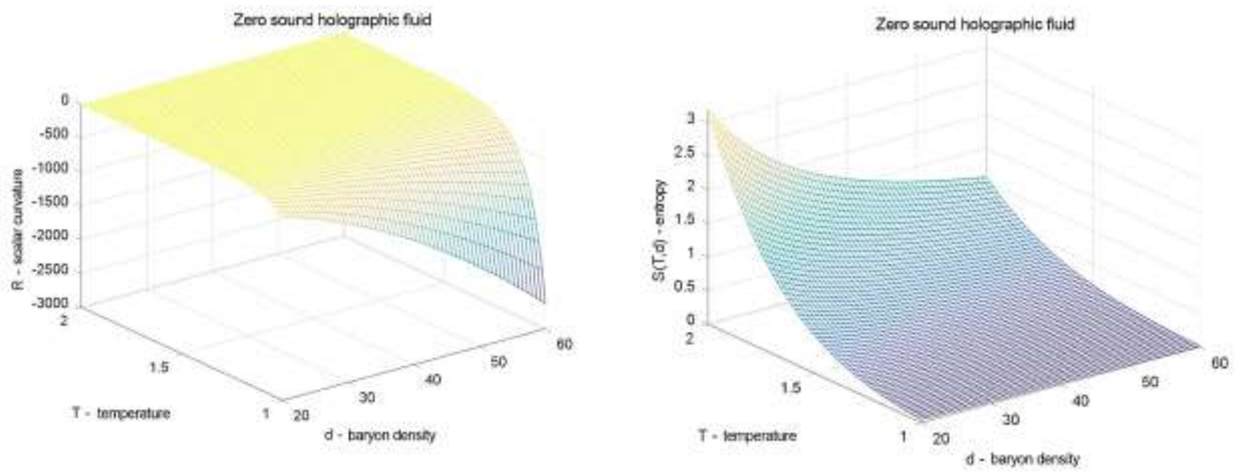
The determinant of this tensor:

$$\det(g)=-\frac{81T^{22}}{d^6} \quad (10)$$

and scalar curvature:

$$R_1=\frac{398\,d^2}{216\,T^{12}}. \quad (11)$$

Formulas (8) and (11) show that the scalar curvature tends to minus and plus infinities when the temperature tends to zero and when the baryon density increases, which indicates a possible phase transition in this region. It can also be seen that scalar curvatures tend to zero when the baryon charge density tends to zero and when the temperature increases, which indicates a weakening of the interaction between the particles in the system. The results obtained for a certain range of parameters *T* and d are shown in figures 2*a* and 2*b*. For this system, both metrics (1) and (2) lead to the same General result regarding the location of singularities for the corresponding curvatures.



a) *b*)

Figure 2. Dependence of scalar curvature on temperature and density of baryons: a) - metric was calculated by formula (1), b) - metric was calculated by formula (2).

Conclusion

In this paper, the equilibrium variety of a strongly interacting quantum system with zero sound predicted by the method of holographic dualities is considered within the framework of the GTD, metric tensors and scalar curvatures are calculated for two possible metric variants. Entropy, which depends on temperature and baryon density, was taken as the thermodynamic potential. 3-dimensional graphs are obtained, which clearly show at which values of thermodynamic variables scalar curvatures tend to infinity or to zero, which indicates possible phase transitions and possible compensation of interactions by quantum effects, respectively. It is shown that both variants of metrics (1) and (2) in this case lead to the same conclusion regarding the location of the lines of possible phase transitions in the considered holographic system with zero sound.

References

1 Maldacena J. The large N limit of super conformal field theories and supergravity //Adv. Theor. Math. Phys. – 1998. – C.231-252.

2 Karch A., Son D.T., and Starinets A.O. Zero sound from holography //Phys. Rev. Lett. – 2009. – Vol.102. – Art.No 051602

3 Karch A., O'Bannon A. Holographic thermodynamics at finite baryon density: some exact results //JHEP 11. – 2007. – Vol.2007. – Art.No 074.

4 Quevedo H. Geometrothermodynamics //J. Math. Phys. – 2007. – Vol. 48. – Art.No 013506.
5 Quevedo H., Sanchez A., Taj S., Vazquez A., Phase transitions in Geometrothermodynamics //Gen. Rel. Grav. – 2011. – Vol. 43. – Art.No 1153.
6 Engelhardt N. and Horowitz G.T. Recovering the spacetime metric from a holographic dual //Adv. Theoretical Math. Phys. – 2017. – Vol. 21. No.7. – P.1635-1653.
7 Czech B., Lamprou L., McCandlish S. and Sully J. Integral geometry and holography //J. High Energy Phys. –2015. –No. 175. – P. 7-33.
8 Bhattacharya J., Veronika E. Hubeny, Mukund Rangamani, Tadashi Takayanagi. Entanglement density and gravitational thermodynamics //Phys. Rev. D. – 2015. – Vol.91. – Art.No 106009.
9 Quevedo H., Sasha A., Zaldivar. A geometrothermodynamic approach to ideal quantum gases and Bose-Einstein condensates //J. General Relativity and Quantum Cosmology. – 2015. arXiv:1512.08755v3.
10 Mansoori, Seyed Ali Hosseini et.al. Extrinsic and intrinsic curvatures in thermodynamic geometry //Phys. Rev. B. – 2016. – Vol. 759. – P.298-305.
11 Ming Zhang, Xin-Yang Wang, Wen-Biao Liu. Extrinsic curvature in geometrothermodynamics //Physics Letters B. – 2018. – Vol.783. – P.169-174.
12 Ming Zhang Corrected thermodynamics and geometrothermodynamics for anti-de Sitter black hole //Nuclear Physics B. – 2018. – Vol.935. – P.170-182.
13 Karch A., O'Bannon A. Holographic thermodynamics at finite baryon density: some exact results //JHEP 11. – 2007. – Vol.2007. – Art.No 074.
14 Yunseok S., Sang-Jin Sin, Jonathan Shock, Dimitrios Zoakos Holographic hadrons in a confining finite density medium //Journal of High Energy Physics. – 2010. – Vol.3. – Art.No 115.
15 Xun Chen, Danning Li, Mei Huang Criticality of QCD in a holographic QCD model with critical end point //Chinese Physics C. -2019. – Vol.43. – No 2. – Art.No 023105.
16 Alsup, James et.al. Duality between zeroes and poles in holographic systems with massles fermions and a dipole coupling //Phys. Rev. D90. – 2014. –No.12. – Art.No 126013.
17 Pineda V., Quevedo H., Maria N. Quevedo, Sanchez A., Valdes E. The physical significance of geometrothermodynamic metrics //J. of Geometric Methods in Modern Physics. – 2019. – Vol.16. –No. 11. – Art.No 1950168.
18 Quevedo H., Nettel F., Bravetti A. Representation invariant Geometrothermodynamics: applications to ordinary thermodynamic systems //J. of Geom. and Physics. -2014. – Vol.81. – P.1-9.
19 Maria N. Quevedo, Quevedo H. Geometrothermodynamics of the cosmological constant //International Journal of Management and Applied Science. – 2017. – Vol.3. – P.21-24.
20 Nikola I. Gushterov, Andy O'Bannon, Ronnie Rodgers Holographic zero sound from Spacetime – filling branes //J. of High Energy Physics. – 2018. – Art.No.076.

References

1 J. Maldacena, Advances Theoretical Math. Phys, 2 (02), 231-252 (1998).
2 A., Karch D.T. Son, Phys. Rev. Lett., 102 (2009).
3 A. Karch, A. O'Bannon, Journal of high energy physics, 11, 074 (2007).
4 H. Quevedo, Journal Math. Phys. 48, 013506 (2007).
5 H. Quevedo, A. Sanchez, S. Taj, A. Vazquez, Gen. Rel. Gravity 43, 1153 (2011).
6 N. Engelhardt and G.T. Horowitz, Advances Theor. Math. Phys. 21(7), 1635-1653, (2017).
7 B. Czech, L. Lamprou, S. McCandlish and J. Sully, J. of High Energy Phys. 175, 7-33 (2015).
8 J. Bhattacharya, E.V. Hubeny, M. Rangamani, T. Takayanagi, Phys.Rev.D, 91, 106009 (2015).
9 H. Quevedo, A. Sasha, S. Zaldivar, J. General Relativity and Quantum Cosmology, arXiv:1512.08755v3 (2015).
10 Mansoori, Seyed Ali Hosseini et.al. Phys. Rev. B, 759, 298-305 (2016).
11 Ming Zhang, Xin-Yang Wang, Wen-Biao Liu, Physics Letters B, 783, 169-174 (2018).
12 Ming Zhang, Nuclear Physics B. 935, 170-182 (2018).
13 Andreas Karch, Andy O'Bannon, J. of High Energy Physics, 11(074) (2007).
14 S. Yunseok, S. Jin Sin, J. Shock, D. Zoakos, Journal of High Energy Physics, 3(115) (2010).
15 Xun Chen, Danning Li, Mei Huang, Chinese Physics C.43(2), 023105 (2019).
16 Alsup, James et.al., Phys. Rev. D, 90(12), 126013 (2014).
17 V. Pineda, H. Quevedo, N. Maria Quevedo, A. Sanchez, E. Valdes, J. of Geometric Methods in Modern Physics, 16(11), 1950168 (2019).
18 H. Quevedo, F. Nettel, A. Bravetti, J. of Geometry and Physics, 81, 1-9 (2014).
19 N. Maria Quevedo, H. Quevedo, International Journal of Management and Applied Science, 3, 21-24 (2017).
20 Nikola Gushterov, Andy O'Bannon, Ronnie Rodgers, J. of High Energy Physics, 76, (2018).