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THE PARTICLE SIZE EFFECT ON THE COLLISION PROCESSES IN COMPLEX PLASMA

In this paper, the impact of dust particle size on the collision processes of ions with dust particles is investigated on the basis of the modified Yukawa potential. The scattering and absorption of an ion in the field of a dust particle with different radius are considered. The scattering angles of ions in the field of a charged dust particle at different impact parameters are calculated. The scattering and absorption cross sections of ions in the field of a dust particle were obtained considering their size in a wide range of coupling parameters. And also, the ion drag force acting on the charged dust particle is investigated. The orbital and collector parts of the ion entrainment force are obtained for different dust particle radii. The results show that taking into account the dust particle radius leads to a sufficient change in both the momentum transfer cross section and the ion entrainment force. It is found that accounting for dust particle size leads to a decrease in the force due to ion scattering and, conversely, to an increase in the force due to ion absorption. The results of this work can be useful for studying wave phenomena, configuration and arrangement of dust structures, rotation of dust clusters in a magnetic field, dispersion of low-frequency oscillations, formation of voids, and other collective processes.

Keywords: complex plasma, dust particle, collision processes, momentum transfer cross section, ion drag force.

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Бөлшектер өлшемінің кешенді плазмадағы соқтығысу процестеріне әсері

Бұл жұмыста Юкаваның модификацияланған потенциалы негізінде тозаңды бөлшектердің өлшемдерін ескере отырып, иондардың тозаңды бөлшектермен соқтығысу процестеріне әсері зерттелді. Радиустары әртүрлі тозаңды бөлшектерінің өрісінде ионның шашырауы мен жұтылуы қарастырылды. Әртүрлі нысана параметрлері үшін зарядталған тозаңды бөлшектерінің өрісіндегі иондардың шашырау бұрыштары есептелді. Тозаңды бөлшек өрісіндегі иондардың шашырау және жұтылу қималары байланыс параметрлерінің кең диапазонында олардың өлшемдерін ескере отырып алынды. Сондай-ақ зарядталған тозаңды бөлшектеріне әсер ететін иондық қарпу күші зерттелді. Тозаңды бөлшектерінің әртүрлі радиустары үшін иондық қарпу күшінің орбиталық және коллекторлық бөліктері алынды. Алынған нәтижелер тозаңды бөлшектерінің радиусын есепке алу импульсті тасымалдау қимасының да, иондық қарпу күшінің де жеткілікті өзгеруіне әкелетінін көрсетеді. Анықталғандай, тозаңды бөлшектерінің мөлшерін есепке алу иондардың шашырауынан болатын күштің төмендеуіне және керісінше иондардың жұтылуынан болатын күштің жоғарылауына әкеледі. Бұл жұмыстың нәтижелері толқындық құбылыстарды, тозаңды құрылымның конфигурациясын және орналасуын, магнит өрісінде тозаңды кластерлердің айналуын, төменгі жиілікті тербелістердің дисперсиясын, вейдтың пайда болуын және басқа да ұжымдық процестерді зерттеу үшін пайдалы болуы мүмкін.

Түйін сөздер: кешенді плазма, тозаңды бөлшектер, соқтығысу процестері, импульсті тасымалдау қимасы, иондық қарпу күші.

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Влияние размера частиц на процессы столкновения в комплексной плазме

В данной работе были исследованы влияние размера пылевых частиц на процессы столкновения ионов с пылевыми частицами на основе модифицированного потенциала Юкавы. Были рассмотрены рассеяние и поглощение иона в поле пылевой частицы с различным радиусом. Были рассчитаны углы рассеяния ионов в поле заряженной пылевой частицы при различных прицельных параметрах. Были получены сечения рассеяния и поглощения ионов в поле пылевой частицы с учетом их размера в широком диапазоне параметров связи. А также была исследована сила ионного увлечения, действующая на заряженную пылевую частицу. Для различных радиусов пылевых частиц были получены орбитальная и коллекторная части силы ионного увлечения. Полученные результаты показывают, что учет радиуса пылевых частиц приводит к достаточному изменению как сечения переноса импульса, так и силы ионного увлечения. Как выяснилось, учет размера пылевых частиц приводит к уменьшению силы, обусловленной рассеиванием ионов, и, наоборот, к увеличению силы, обусловленной поглощением ионов. Результаты данной работы могут быть полезны для изучения волновых явлений, конфигурации и расположения пылевых структур, вращения пылевых кластеров в магнитном поле, дисперсии низкочастотных колебаний, образования войдов и других коллективных процессов.

Ключевые слова: комплексная плазма, пылевая частица, процессы столкновения, сечение переноса импульса, сила ионного увлечения.

Introduction

A number of the fundamental problems concerning dusty plasma is currently in the focus of experimental [1–3] and theoretical [4, 5] studies in laboratory room gas discharge, astrophysical plasmas, as well as various technological processes [6]. Collision processes between charged plasma particles are crucial for examining the transport characteristics of complex plasmas.

The ion drag force resulting from collisions between charged plasma particles influences wave phenomena, the configuration and arrangement of dust structures, the rotation of dust clusters in magnetic fields, the dispersion of low-frequency oscillations, the formation of voids, and other collective processes [7–12].

In studying collisional processes, understanding the interaction between charged particles is crucial. Currently, many studies focus on collisional processes based on the Yukawa potential, where the size of the dust particle is not considered.

In this paper, on the basis of the modified Yukawa potential the collision processes of positively charged ions with the dust particles have been investigated considering their size. The collision process is characterized by the following dimensionless parameters: the coupling parameter $\beta = e^2 Z_d / m v^2 \lambda$, the impact parameter ρ / λ and the radius of the dust particle normalized to the Debye length a / λ .

Theory of interaction

This paper explores the collision processes between a positively charged ion and a dust particle using a modified Yukawa interaction potential that incorporates the effect of the dust particle's finite size [13]:

$$U(r) = -\frac{e^2 Z_d}{(1 + a k_D) r} \exp(-(r - a) k_D), \quad (1)$$

where r is the interparticle distance, k_D is screening length, a is the dust particle radius. In case $a \rightarrow 0$, the modified Yukawa potential (1) corresponds to the Yukawa interaction potential

$$U(r) = -e^2 Z_d \exp(-r k_D) / r.$$

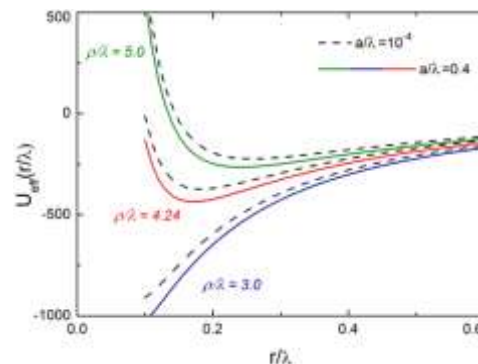


Figure 1 – The effective potential (2) for the radial motion of ions in the field of the dust particle with radius $a/\lambda = 10^{-4}$ and $a/\lambda = 0.4$ at $\rho/\lambda = 3.0; 4.24; 5.0$

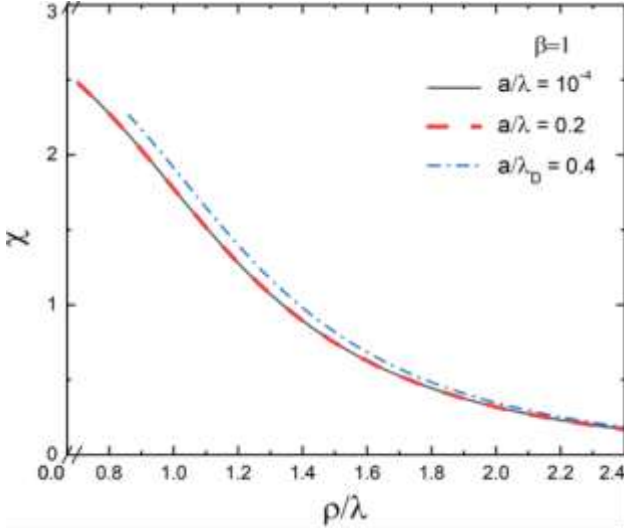
Collision processes

A. Scattering

Let's examine the collision between a positively charged ion and a stationary charged dust particle. The ion's motion within the central force field of the charged dust particle is described by the effective interaction potential energy U_{eff} , given by: [14-16]:

$$\frac{U_{\text{eff}}(r, \rho)}{E} = \frac{\rho^2}{r^2} + \frac{U(r)}{E} \quad (2)$$

where $E = mv_{\infty}^2/2$ represents the initial kinetic energy of the projectile (before the collision), and ρ is the impact parameter. The first term on the right side of equation (2) arises from the centrifugal repulsive force [14-16]. The following equation gives the scattering angle of the projectile based on the impact parameter:



$$\chi(\rho) = |\pi - 2\varphi(\rho)| \quad (3)$$

where $\varphi(\rho) = \rho \int_{r_0}^{\infty} \frac{dr}{r^2 \sqrt{1 - U_{\text{eff}}(r, \rho)}}$.

The scattering cross section can be calculated using the scattering angle (3) in the following manner [25]:

$$\sigma_s = 2\pi \int_0^{\infty} (1 - \cos \chi(\rho)) \rho d\rho \quad (4)$$

The calculation considers the physical condition that r_0 is greater than a . Let us discuss the obtained results of scattering processes of ion in the field of charged dust particle with taking into account its size. The effective potential (2) the radial motion of ions in the field of the dust particle with radius $a/\lambda = 10^{-4}$; 0.4 at $\beta = 100$ and different impact parameters $\rho/\lambda = 3.0$; 4.24, 5.0 are shown in figure 1.

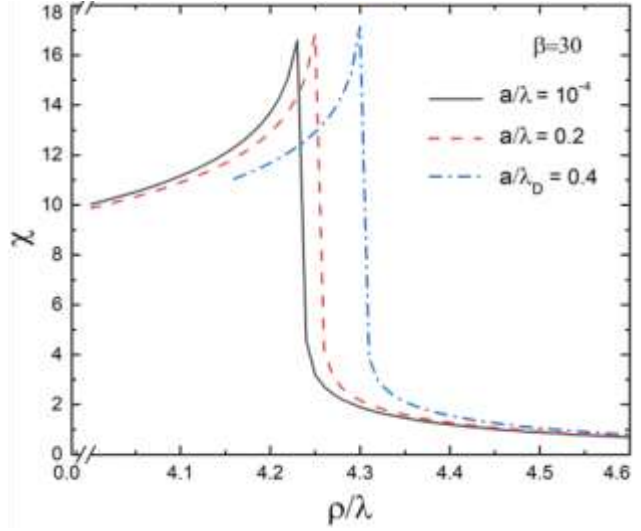


Figure 2 – The dependence of scattering angle on impact parameter for particles of different sizes at $\beta = 1$ and $\beta = 30$

In figure 2 the dependence of scattering angle on impact parameter for particles of different sizes at $\beta = 1$ and $\beta = 30$ are shown. As can be seen from the figures, the influence of dust particle size is enhanced with the increase of the coupling parameter β , i.e. at strong interaction of the ion with the dust particle $\beta \gg 1$.

As previously discussed in the works [15,16], the manifestation of the effect is observed where finite motion of the ion is observed in the field of a dust particle with a large scattering angle at a critical impact parameter $\rho_*/\lambda = 4.24$ and $\beta = 30$ for $\alpha = 10^{-4}$. This effect is preserved, with a small shift in the critical impact parameter $\rho_*/\lambda = 4.27$ for $\alpha = 0.2$ and $\rho_*/\lambda = 4.33$ for $\alpha = 0.4$.

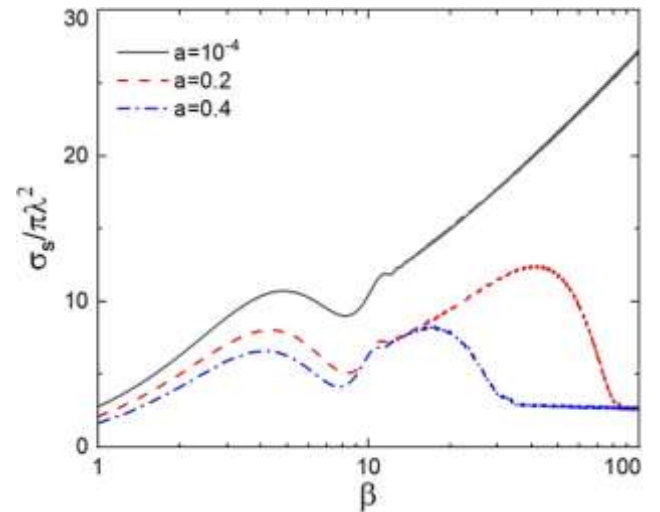


Figure 3 – The dependence of scattering cross section on the coupling parameter for particles of different sizes

Further the scattering cross section (4) has been obtained as a function of the coupling parameter for dust coupling parameter for dust particles of different sizes, which is presented in figure 3. The results show that the consideration of dust particle size leads to a significant reduction of ion scattering around the dust particle. It can also be seen that the number of scattered ions almost becomes negligible from the coupling parameter $\beta > 100$ for $\alpha = 0.2$ and $\beta > 40$ for $\alpha = 0.4$.

B. Capturing

The contact collision cross section is determined from the capture collision (contact collision) parameter $\rho_c(\beta)$, which corresponds to the impact parameter at which the distance of closest approach ($r_{\min}$) corresponds to the contact of the projectile with the dust particle surface [17, 18]. It is assumed that any projectile (ion) with $\rho < \rho_c$ is considered to be captured by a charged dust particle. By knowing $\rho_c(\beta)$ we can calculate the scattering cross section as [17]:

$$\sigma_c(\beta) = \pi \rho_c^2(\beta). \quad (5)$$

The parameter ρ_c is determined by the condition that the closest approach distance, r_0 , is less than the radius of the dust particle, a . This condition ensures an accurate representation of the particle interaction. The closest approach distance (r_0) is computed from the relation $U_{eff}(r, \rho) = 1$. The dependence of collection cross section on the coupling parameter for dust particles of different sizes are given in figure 4. As expected, accounting for dust particle size resulted in an increase in the collection cross section. In figure 5 are shown the dependence of momentum transfer cross section on the coupling parameter for dust particles of different sizes. The momentum transfer cross section is the sum of scattering and collection cross sections. The figure shows that up to $\beta \leq 10$, the weak interaction, i.e., the energy of the colliding particle (ion) is high enough to overcome the potential barrier of the dust particle. In this case, there is mostly scattering of the ion in the field of the dust particle compared to the absorption of the ion by the dust particle. Thus, this leads to a decrease of the total transport cross section as well as the scattering cross section. At strong interaction when $\beta > 10$ the absorption process begins to dominate, thus increasing the total transport cross section.

Ion drag force

The total ion drag force consists of the collection force F_d^{coll} from ions impacting the dust particle and the orbital force F_d^{orb} from deflected ions

$$F_d = F_d^{orb} + F_d^{coll}, \quad (6)$$

The orbital component of the ion drag force at drift velocity u is calculated using an integral over the shifted Maxwell distribution [19]:

$$F_d^{coll}(u) = \frac{2^{1/2} n_i m_i}{\pi^{1/2} v_{Ti} u} \int_0^\infty \sigma(v) v^3 \exp\left(-\frac{(v-u)^2}{2v_{Ti}^2}\right) \left(1 - \exp\left(-\frac{2uv}{v_{Ti}^2}\right) - \frac{v_{Ti}^2}{2uv} (1 - \exp\left(\frac{2uv}{v_{Ti}^2}\right))\right) dv \quad (7)$$

where, $v_{Ti} = (k_B T_i / m_i)^{1/2}$ represents the ion thermal velocity. The collection component of the ion drag force is obtained from the following equation [20]:

$$F_d^{coll} = n_i v_s m_i v_i \pi \rho_c^2 \quad (8)$$

where, $v_s = (v_i^2 + 8\kappa_B T_i / (\pi m_i))^{1/2}$ is the ion thermal speed.

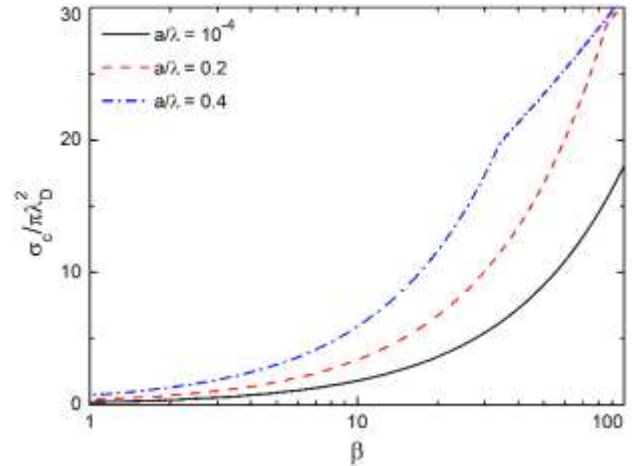


Figure 4 – The dependence of collection cross section on the coupling parameter for dust particles of different sizes

The dependence of ion drag force on the velocity for dust particles of different sizes is shown in figure 6. In figure the total ion drag force (6) (solid line), orbital part (7) (dash dot line) and collection part (8) (dash line) of the ion drag force are presented for different values of dust particle radius. The results presented in figure 6 were obtained using the cross-sections calculated for the potential (1). As can be seen from the figure, taking into account the particle size leads to a reduction of the orbital part of the ion drag force, while the collection part of the ion drag force conversely grows.

For Argon plasma with the density of electrons and ions $5 \times 10^{-15} \text{ m}^{-3}$, the temperature of electrons (ions) $T_e = 3 \text{ eV}$ ($T_i = 0.03 \text{ eV}$), and the dust particle radius $a = 3.62$ ($a/\lambda = 0.2$), $a = 7.25$ ($a/\lambda = 0.4$) corresponds to the charge $Z_d = 5735$ ($Z_d = 11484$).

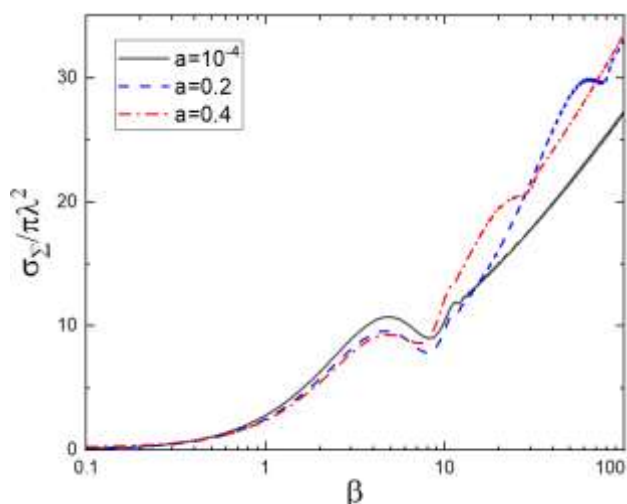


Figure 5 – The dependence of momentum transfer cross section on the coupling parameter for dust particles of different sizes

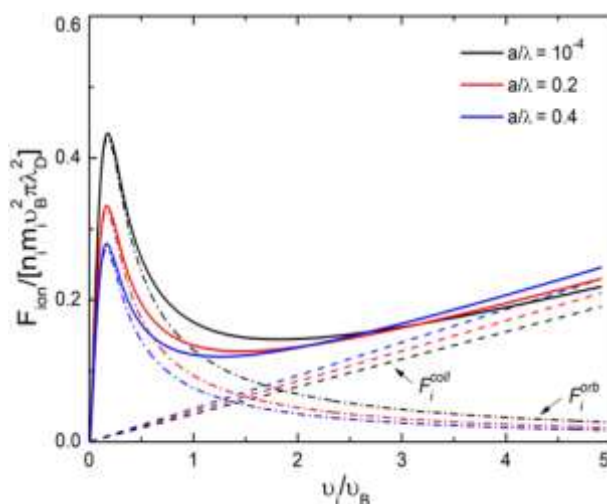


Figure 6 – The dependence of ion drag force on the velocity for dust particles of different sizes. The dash line corresponds to the collection component of the ion drag force

Conclusions

The collision processes of charged particles have been investigated on the basis of the modified Yukawa potential considering the dust particle radius. The scattering as well as the capturing of ion in the field of charged dust particle with different radius are considered. The orbital and collection parts of ion drag force are obtained on the basis of the cross sections for different dust particle radius in a wide range of the coupling parameter. Accounting the dust particle radius led to a sufficient change for both the

momentum transfer cross section and the ion drag force. As it is found out, taking into account the size of dust particles leads to a reduction of the force due to scattering ions and on the reverse increases the force due to absorbing ions.

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