IRSTI 29.35.17 https://doi.org/10.26577/RCPh20259439

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UNRAVELING TRAFFIC JAM COMPLEXITY: INSIGHTS FROM ELEMENTARY CELLULAR AUTOMATA AND STATISTICAL PHYSICS

The Nagel-Schreckenberg (NaSch) model is a foundational cellular automaton framework for modeling highway traffic dynamics. Despite its simplicity, the model reproduces key features of real traffic, including the spontaneous emergence of stop-and-go waves and phantom jams. This study investigates the critical transition from free-flow to congested traffic as vehicle density increases. Through extensive simulations on large systems, we analyze steady-state observables such as mean velocity, traffic flow, velocity variance, and the number of jammed clusters. A sharp transition is observed in both microscopic and macroscopic indicators, with velocity fluctuations peaking near a critical density that signifies the onset of jamming. To interpret this behavior, we develop a mean-field theoretical framework that predicts the critical transition point based on vehicle interactions and stochastic braking. The theoretical prediction closely matches the numerical location of the peak in velocity variance. Additionally, we study the formation and coalescence of jams, identifying a secondary peak in jam count at higher densities, where small clusters merge into large-scale gridlock. These findings provide a quantitative understanding of the jamming transition in traffic flow and highlight the relevance of nonequilibrium phase transition theory in transportation systems. The NaSch model thus serves as a paradigmatic example of how simple local rules can give rise to emergent collective phenomena and critical behavior.

Key words: statistical physics, traffic flow, phase transition.

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Көліктің кептелісін ашу кешені: элементар торлы автоматика және статистикалық физика тәсілдері көзғарасымен

Нагель–Шреккенберг (NaSch) моделі автомобиль жолдарындағы қозғалыс динамикасын сипаттайтын жасушалық автоматтың іргелі моделі болып табылады. Қарапайымдылығына қарамастан, бұл модель нақты қозғалыстың негізгі ерекшеліктерін, соның ішінде өздігінен пайда болатын «тоқтау-жүру» толқындарын және фантомдық кептелістерді дәл бейнелейді. Бұл зерттеуде көлік тығыздығы артқан сайын еркін ағымнан кептеліске өту кезіндегі сыни ауысу қарастырылады. Ірі жүйелерде жүргізілген кең ауқымды модельдеулер нәтижесінде орташа жылдамдық, қозғалыс қарқындылығы, жылдамдықтың дисперсиясы және кептеліс кластерлерінің саны сияқты стационарлық шамалар талданды. Микродеңгейде де, макродеңгейде де айқын ауысу байқалады, ал жылдамдықтың флуктуациялары кептеліс басталатын сыни тығыздық маңында шегіне жетеді. Бұл құбылысты түсіндіру үшін көліктердің өзара әрекеттесуіне және кездейсоқ тежелуіне негізделген орташа өрістік теориялық тәсіл әзірленді. Теориялық болжам жылдамдық дисперсиясының шыңы байқалатын сандық нәтижелермен жақсы сәйкес келеді. Сонымен қатар, кептелістердің пайда болуы мен бірігуі зерттелді: жоғары тығыздықта ұсақ кластерлердің ірі көлемді қозғалыс тоқырауына бірігуімен байланысты кептеліс санының екінші шыңы анықталды. Бұл нәтижелер көлік ағынындағы кептелістердің пайда болу ауысуын сандық тұрғыда түсінуге мүмкіндік береді және көлік жүйелерін зерттеуде тепе-теңсіз фазалық ауысулар теориясының өзектілігін көрсетеді. Осылайша, NaSch моделі қарапайым жергілікті ережелердің ұжымдық құбылыстар мен сыни мінез-құлықты тудыруының парадигмалық мысалы бола алады.

Түйін сөздер: статистикалық физика, қозғалыс ағыны, фазалық ауысу.

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Раскрытие сложности дорожных пробок: взгляд с точки зрения элементарных клеточных автоматов и статистической физики

Модель Нагеля–Шреккенберга (NaSch) является фундаментальной клеточной автоматной моделью для описания динамики дорожного движения на автомагистралях. Несмотря на простоту, модель воспроизводит ключевые особенности реального трафика, включая спонтанное возникновение волн «стоп-и-го» и фантомных пробок. В данной работе исследуется критический переход от свободного движения к заторам при увеличении плотности транспортного потока. На основе обширных численных экспериментов на больших системах анализируются стационарные наблюдаемые величины, такие как средняя скорость, интенсивность движения, дисперсия скоростей и количество заторных кластеров. В обоих случаях — на микроскопическом и макроскопическом уровнях — наблюдается резкий переход, при этом флуктуации скорости достигают максимума вблизи критической плотности, указывая на начало формирования пробок. Для интерпретации этого поведения разработан теоретический среднеполевой подход, предсказывающий критическую точку перехода на основе взаимодействия транспортных средств и стохастического торможения. Теоретический прогноз хорошо совпадает с численным положением пика дисперсии скоростей. Кроме того, исследуется формирование и слияние заторов, при котором на более высоких плотностях наблюдается второй пик числа пробок, связанный с объединением малых кластеров в крупномасштабный застой. Эти результаты дают количественное понимание перехода к пробкам в транспортных потоках и подчеркивают значимость теории фазовых переходов в неравновесных системах для транспортных исследований. Таким образом, модель NaSch служит парадигматическим примером того, как простые локальные правила могут порождать коллективные явления и критическое поведение.

Ключевые слова: статистическая физика, транспортный поток, фазовый переход.

Introduction

Understanding the emergence of traffic congestion from simple local interactions remains a central problem in the theory of driven many-particle systems. Among the most influential models in this area is the Nagel–Schreckenberg (NaSch) model, a minimal cellular automaton introduced by Nagel and Schreckenberg in 1992 [1]. Despite its simplicity, the model captures essential features of vehicular dynamics on highways, including spontaneous jam formation, stop-and-go waves, and the characteristic fundamental diagram relating vehicle flow to density.

The NaSch model represents a road as a one-dimensional periodic lattice, where each cell is either empty or occupied by a car with discrete velocity $v \in \{0,1,\ldots,v_{\max}\}$. The dynamics proceeds in parallel updates governed by simple rules: acceleration, braking due to the gap ahead, stochastic deceleration (with probability p), and movement. These rules are sufficient to generate a transition from a free-flow regime at low density to a congested regime at high density, even in the absence of bottlenecks. This phenomenon, known as phantom jamming, reflects a self-organized instability arising from the nonlinear interplay between local interactions and noise.

Theoretical investigations of this transition have explored both equilibrium and nonequilibrium perspectives. A recent study by Jha et al. [2, 11, 12] demonstrated that the NaSch model exhibits signatures of nonequilibrium criticality near the jamming transition, including diverging relaxation times and enhanced fluctuations. These features parallel classical second-order phase transitions, although the NaSch model operates far from thermodynamic equilibrium. To estimate the critical point analytically, mean-field and cluster approximations have been employed [3, 4].Further refinements using cluster mean-field theory or kinetic approaches have attempted to incorporate spatial correlations more accurately [4, 5, 6, 16, 17]. Such models reveal that near the transition point, traffic exhibits metastability, hysteresis, and spatially intermittent structures, consistent with empirical traffic data [7, 8, 13-15, 20].

Recent work on entropy production and broken detailed balance in active systems also offers a new framework to quantify the irreversibility and information flow in jammed traffic [9, 10, 18, 19]. These perspectives extend beyond classical traffic theory, connecting jamming transitions to broader principles in nonequilibrium statistical mechanics.

In summary, the NaSch model provides a paradigmatic platform for studying emergent jamming phenomena in driven systems. The critical transition from free flow to congestion can be located both numerically – via peaks in velocity variance or jam counts – and theoretically – via mean-field analysis. These insights are relevant not only for traffic engineering but also for understanding the universality of dynamical phase transitions in active, stochastic systems.

Nagel-Schreckenberg model

The study of vehicular traffic has long fascinated both physicists and engineers, for it represents one of the most visible and economically important instances of a self-organized system. Despite the apparently mundane nature of cars moving along a road, the collective dynamics that emerge: traffic jams, stop-and-go waves, and metastable states, display all the richness of a complex system governed by nonlinear interactions and stochastic influences. In the early 1990s, Kai Nagel and Michael Schreckenberg introduced a minimal yet remarkably powerful model for simulating traffic flow, which has since become a cornerstone of modern traffic theory: the Nagel–Schreckenberg (NaSch) cellular automaton model [12].

At its heart, the Nagel–Schreckenberg model embraces the philosophy of parsimony. Instead of attempting to replicate every microscopic detail of driving behavior, it reduces the highway to its bare essentials: a one-dimensional array of discrete sites representing road cells, and vehicles that occupy these sites while obeying a set of simple rules. Each car is characterized only by itsposition and velocity, measured in integer units. Time advances in discrete steps, and all cars update their state simultaneously according to a handful of update rules. These rules encode the essence of acceleration, braking to avoid collisions, random fluctuations in driver behavior, and the actual forward motion of the vehicles. What is striking is how much macroscopic realism emerges from this deceptively simple microscopic scheme.

The lattice structure of the model consists of L cells arranged in a ring or along an open road. Each cell can be either empty or occupied by a single vehicle. If there are N cars on the lattice, the global density is $\rho = N/L$. Each car i has a velocity v_i, constrained to lie between zero and some maximum value $v_{\max}$. The state of the system at a given time step is thus specified by the set of all positions and velocities $\{x_i, v_i\}$. The key to the Nagel–Schreckenberg model lies in the update rules, which are applied synchronously to every vehicle at each time step:

(i) **Acceleration**. If a car is not yet traveling at the maximum velocity, it attempts to increase its speed by one unit. This simple rule captures the natural tendency of drivers to accelerate whenever possible, reflecting the desire to travel as fast as conditions allow.

(ii) **Deceleration due to other cars**. If the distance (gap) to the next car ahead is smaller than the current velocity, the car reduces its velocity to avoid collision. This encodes the essential constraint of car-following: no vehicle can advance further than the empty space in front of it.

(iii) **Randomization**. With a given probability p, each car reduces its velocity by one unit, provided it is greater than zero. This step models the stochastic elements of human driving: hesitation, imperfect reactions, distractions, and introduces fluctuations into the system that play a crucial role in generating realistic traffic phenomena.

(iv) **Movement**. Finally, each car advances forward by a number of cells equal to its updated velocity. This step simply executes the motion determined by the previous rules.

These four steps: acceleration, braking, randomization, and movement, are the entirety of the Nagel–Schreckenberg algorithm. Their appeal lies not only in their simplicity but also in their synchronous application: all vehicles update simultaneously, ensuring that no artificial priority is assigned to any driver. Despite the minimalism of the rules, the emergent behavior of the system captures many of the empirically observed features of highway traffic.

One of the first observations made by Nagel and Schreckenberg was that the model exhibits a sharp transition between free flow and congested traffic. At low densities, cars accelerate to the maximum velocity, and traffic proceeds smoothly with minimal interactions. As density increases, interactions between vehicles become more frequent, leading to the appearance of localized slowdowns. Beyond a critical density, the system undergoes a phase transition: stable traffic jams form, propagating backwards through the system as stop-and-go waves. These jams are not artifacts of the discretization; rather, they mirror the phantom traffic jams observed on real highways, which arise spontaneously without any visible obstacle.

The introduction of the randomization step is crucial for the appearance of such phenomena. Without it, the system would behave deterministically, with cars forming perfectly ordered platoons that unrealistically eliminate many features of real traffic. The stochastic element ensures that minor fluctuations amplify into large-scale congestion under conditions of high density. In this way, the Nagel–Schreckenberg model highlights the dual role of randomness: at once a source of noise and an essential mechanism for the emergence of collective patterns.

A further strength of the model is its ability to reproduce the so-called fundamental diagram of traffic flow: the relationship between the global density of vehicles and the average flow (vehicles per unit time). At low densities, the flow increases linearly with density, as more cars occupy the road while still moving near the maximum velocity. After reaching an optimal density, however, the flow begins to decline as congestion dominates and jams suppress movement. The resulting diagram has the characteristic concave shape observed in empirical traffic measurements. Importantly, the maximum flow, corresponding to the capacity of the road, depends sensitively on the randomization probability p and the maximum velocity $v$max, thus linking the microscopic rules to macroscopic transport capacity.

Over the years, the Nagel–Schreckenberg framework has been extended in multiple directions. Multi-lane versions have been developed to model overtaking and lane-changing maneuvers, incorporating probabilistic rules for lateral movement. Variants introduce heterogeneous driver populations, with differing maximum velocities or braking behaviors, to reflect the diversity of vehicles and drivers. Other extensions include open boundary conditions with inflow and outflow of cars, modeling real highways rather than closed loops; inclusion of traffic lights and intersections; and adaptations to study pedestrian motion. Each of these elaborations preserves the cellular automaton spirit while enhancing realism for specific applications.

The model’s success is not limited to qualitative realism. It has been employed in large scale simulations of real traffic networks, where its computational efficiency is particularly advantageous. Unlike continuous car-following models that rely on differential equations, the Nagel–Schreckenberg automaton updates millions of vehicles using simple integer arithmetic, making it ideal for large-scale urban or national traffic forecasts. Indeed, it formed the basis for some of the earliest attempts to use parallel computing for simulating traffic in metropolitan areas, demonstrating its enduring utility beyond theoretical explorations.

From a broader scientific perspective, the Nagel–Schreckenberg model exemplifies the power of minimal models in statistical physics. By stripping away extraneous detail, it isolates the essential mechanisms that govern macroscopic behavior: acceleration toward a preferred velocity, deceleration due to interactions, stochastic perturbations, and conservation of spatial exclusion. This reductionist approach is reminiscent of models like the Ising model in magnetism: while the rules are simple and unrealistic in detail, the emergent patterns capture the critical essence of the phenomenon under study. In this sense, the NaSch model is to traffic flow what the Ising model is to ferromagnetism, a paradigmatic example that guides both theoretical understanding and applied research.

Critics have noted that the discrete nature of the model, cars jumping between cells and velocities changing in integer steps, may oversimplify the continuous character of real driving. Yet such criticism misses the point: the NaSch model was never meant as an engineering tool for designing highways with millimeter precision. Rather, it is a conceptual and computational laboratory for exploring the fundamental laws of traffic dynamics. The fact that it reproduces empirical flow–density relations, spontaneous jam formation, and metastable states with such frugality of assumptions speaks to the robustness of its underlying principles.

In conclusion, the Nagel–Schreckenberg model stands as one of the most influential contributions to traffic theory in the past three decades. Its cellular automaton formulation distills the essence of driving into a handful of synchronous rules, yet yields an astonishing variety of emergent phenomena that mirror reality. It bridges the microscopic scale of individual drivers with the macroscopic scale of traffic waves and flow diagrams, providing both conceptual clarity and computational efficiency. Beyond traffic, it has inspired analogous models in domains as varied as pedestrian dynamics, biological transport, and granular flow. For physicists and engineers alike, it remains a model case of how simplicity, when chosen wisely, can illuminate the complexity of the world.

Critical jam transition

A powerful tool for analyzing the emergent behavior of the NaSch cellular automaton is the space–time diagram, exemplified in Figure 1. In this representation, the abscissa denotes the discrete spatial sites of the roadway, while the ordinate indicates the progression of discrete time steps. Vehicle positions are plotted as black dots, such that the trajectory of each car is represented by a sequence of points forming an approximately diagonal line.

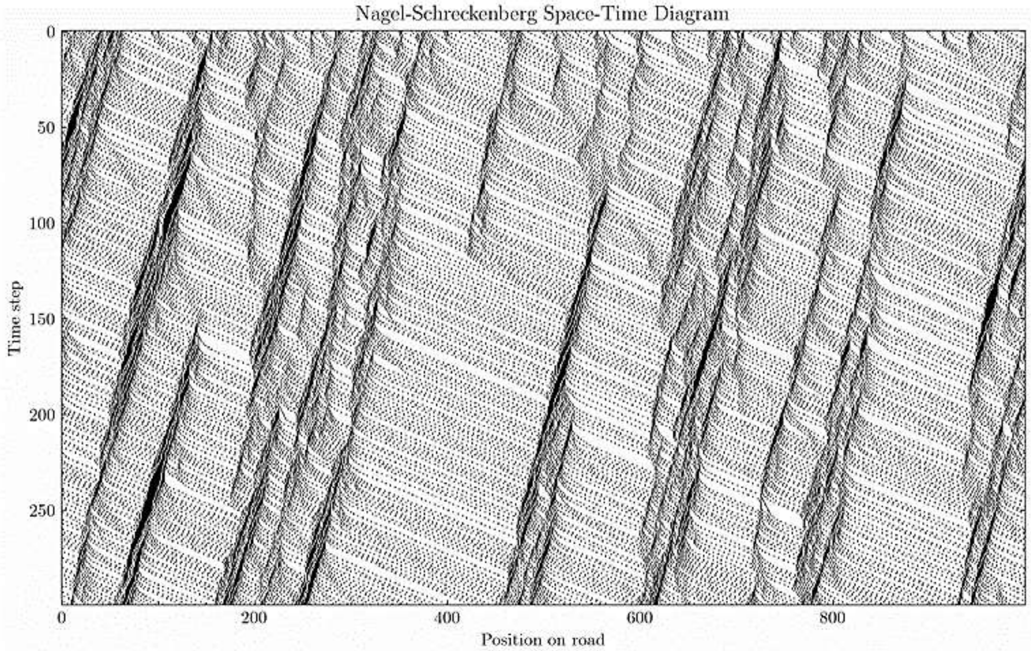

Figure 1 – Space–time diagram of the Nagel-Schreckenberg cellular automaton model. The horizontal axis represents position along the discretized road, while the vertical axis denotes discrete time steps (increasing downward). Black dots correspond to occupied cells, and diagonal lines trace the trajectories of individual vehicles. Dense, backward-propagating diagonal bands indicate spontaneously emerging traffic jams, while regions of shallow, nearly parallel lines correspond to free flow. The diagram illustrates the characteristic formation of stop-and-go waves arising from the microscopic update rules of the model.

In regimes of free flow, trajectories appear as nearly parallel straight lines with slopes inversely proportional to vehicle velocities. Shallow slopes correspond to high velocities, whereas steep slopes indicate reduced motion. Deviations from linearity occur when interactions between vehicles take place: a faster vehicle approaching a slower predecessor is forced to decelerate, producing a distinct kink in the trajectory. The stochastic braking rule of the NaSch model further induces small fluctuations, leading to the irregular microscopic patterns visible in the diagram.

The most salient structures in Figure 1 are the dense diagonal bands extending from top left to bottom right. These bands correspond to traffic jams, i.e., spatiotemporal regions of high vehicle density and low average velocity. Notably, these jams propagate backwards relative to the direction of vehicle flow, a phenomenon consistently observed in empirical highway data. The process is cyclic: vehicles enter a jam from the right, experience prolonged deceleration or standstill, and subsequently accelerate again upon exiting to the left. Between jammed domains, trajectories exhibit free-flow characteristics.

Such diagrams reveal the spontaneous formation of stop-and-go waves: collective oscillatory modes of congestion that arise in the absence of external perturbations or bottlenecks. Hence, the space–time representation demonstrates how macroscopic traffic patterns, including metastability, wave propagation, and flow breakdown – emerge directly from the simple microscopic interaction rules of the NaSch model.

The NaSch model is a prototypical cellular automaton for studying traffic flow. It exhibits a transition from free flow to congestion as vehicle density increases. To estimate the critical density of this transition analytically, we employ a mean-field approach by modeling the system as a one-dimensional lattice where each site is either empty (with probability $1-\rho$) or occupied by a vehicle with velocity v, occurring with probability c_v. The overall car density is $\rho = \sum_{v=0}^{v_{max}} c_v$.

Assuming spatial independence between cells, the probability distribution of gaps g between cars becomes geometric: $(g) = (1-\rho)^g$. The mean gap is then $\langle g \rangle = \frac{1-\rho}{\rho}$, and the mean headway is $h = \frac{1}{\rho}$. Car dynamics follow four rules: acceleration, braking (by gap), random deceleration (with probability p), and movement.

In the free-flow regime, vehicles never brake due to interactions and only slow down due to randomization. This holds when $\langle g \rangle > v_{max}$, leading to the condition $\rho < \frac{1}{v_{max}+1}$. The average velocity becomes $v_{free} = v_{max} - p$, yielding a free-flow flux $J_{free} = (v_{max} - p)$.

In the interaction-limited regime, where $\rho > \frac{1}{v_{max}+1}$ vehicles frequently brake due to insufficient headway. Here, the system flux is approximated as $J_{jam} = 1-\rho$, corresponding to the outflow from jammed clusters.

The full mean-field fundamental diagram is constructed as $(\rho) = \min[\rho(v_{max} - p), 1-\rho]$, and the critical density corresponds to the intersection point of the two branches:

$$\rho_c = \frac{1}{1 + v_{max} - p}.$$

This expression predicts the onset of congestion and agrees well with numerical simulations in the large-system limit.

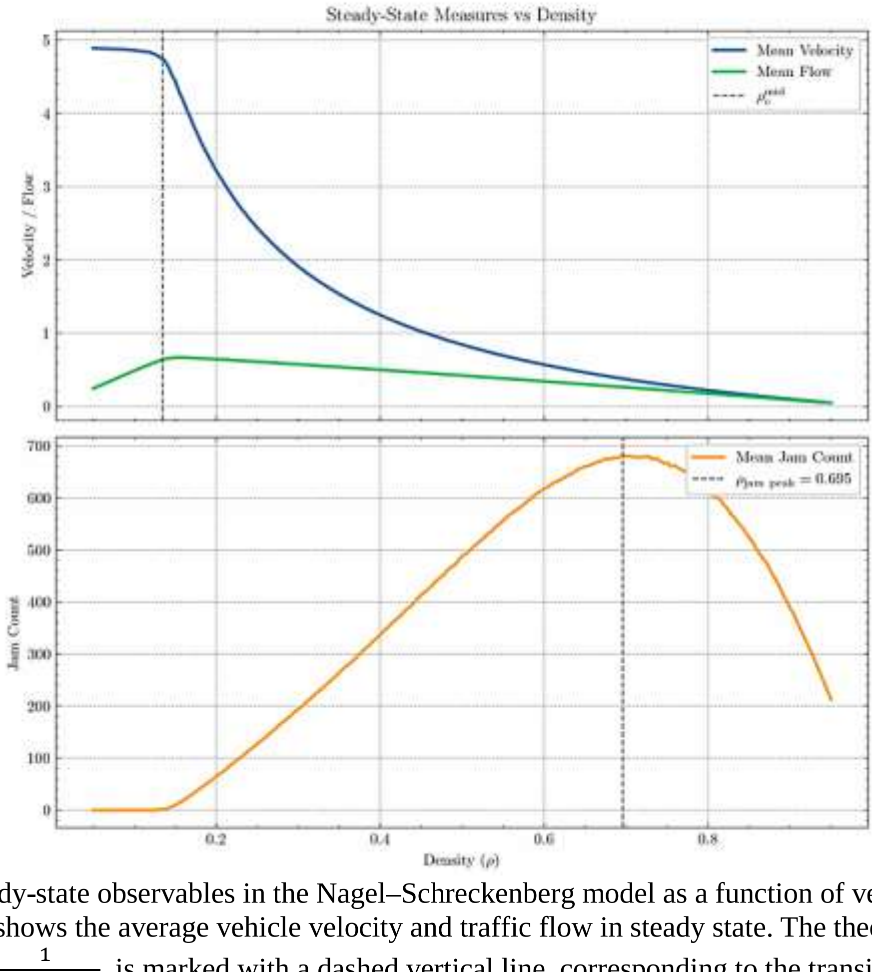

Figure 2 – Steady-state observables in the Nagel–Schreckenberg model as a function of vehicle density ρ. The top panel shows the average vehicle velocity and traffic flow in steady state. The theoretical critical density $\rho_c^{mid} = \frac{1}{1+v_{max}-p}$ is marked with a dashed vertical line, corresponding to the transition predicted by mean-field theory. The bottom panel displays the mean number of jammed clusters (defined as contiguous segments of vehicles with velocity zero). The peak jam count occurs at a higher density ρjam, indicating a secondary congestion regime where stop-and go waves coalesce into long-lasting jams. The data was obtained by running NaSch simulations on a periodic lattice of length $L = 5000$, averaging over multiple realizations and long temporal windows to ensure statistical convergence.

Conclusions

The NaSch model remains one of the simplest and most powerful models for investigating the collective behavior of traffic flow. Despite its minimal set of rules are acceleration, stochastic braking, and deterministic motion, the model captures a wide range of realistic traffic phenomena, including spontaneous jam formation, stop-and-go waves, and capacity drop. Central to its significance is the ability to characterize the transition from free flow to congested traffic, which emerges purely from local interactions and noise.

In this study, we examined this transition both numerically and analytically. Through large scale simulations and density sweeps, we computed macroscopic observables such as the average velocity, traffic flow, velocity variance, and jam count. A distinct transition was observed: at low densities, vehicles moved nearly freely, while increasing density led to increased interactions, sharp drops in velocity, and the emergence of jamming. The velocity variance served as a sensitive indicator of this transition, peaking sharply at a critical density.

To explain this behavior, we employed a mean-field theoretical approach, deriving a selfconsistent estimate for the critical density at which free flow becomes unstable. Under the assumption of spatial independence, the critical density was obtained analytically as (3.1). This result aligns well with the onset of strong velocity fluctuations in simulation, confirming its validity in the thermodynamic limit.

Overall, the NaSch model exemplifies how complex macroscopic behavior—such as phase transitions—can arise from simple local rules. The derived critical density formula provides a clear and intuitive understanding of how system parameters govern the stability of traffic flow, and serves as a foundational benchmark for more refined models of vehicular dynamics.

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Мақала тарихы:
Түсті – 04.06.2025
Түзетілген түрде түсті – 21.06.2025
Қабылданды – 20.08.2025

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Article history:
Received 4 June 2025
Received in revised form 21 June 2025
Accepted 20 August 2025

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