









T. Konysbayev^{1,2} , Ye. Kurmanov^{1,2*} , U. Nurlanbek² , G. Rabigulova^{1,2} ,
M. Adil² , B. Bekmurat² , G. Tuzen² , A. Urazalina^{1,2,3} 

¹National Nanotechnology Laboratory of Open Type, Almaty, Kazakhstan

²Al-Farabi Kazakh National University, Almaty, Kazakhstan

³Institute of Nuclear Physics, Almaty, Kazakhstan

*e-mail: kurmanov.yergali@kaznu.kz

QUASI-PERIODIC OSCILLATIONS AROUND COMPACT OBJECTS WITHIN THE SEN SPACETIME

We investigate kilohertz (kHz) quasi-periodic oscillations (QPOs) observed in eight neutron-star low-mass X-ray binaries within the framework of the relativistic precession model (RPM). Fundamental (epicyclic) frequencies of test particles on circular orbits are computed in the static Sen spacetime. By fitting the Keplerian and epicyclic frequencies to the observed lower and upper QPO frequency pairs, (f_L, f_U) , we infer the masses and electric charges of the compact objects and compare the results with those obtained in the Schwarzschild spacetime using the Akaike and Bayesian information criteria (AIC/BIC). We find that the Schwarzschild geometry provides physically consistent fits for four sources, while for GX 5–1 and GX 340+0 the Sen spacetime becomes effectively indistinguishable from Schwarzschild, indicating that no electric charge is required. Although the Sen metric yields statistically improved fits for the remaining four sources, the inferred large masses as well as large electric charges are incompatible with neutron-star physics. We therefore conclude that the static Sen spacetime does not provide a physically viable description of kHz QPOs in neutron-star systems.

Keywords: accretion discs, QPOs, black holes, neutron star, X-ray binaries, relativistic precession model.

Т. Қонысбаев^{1,2}, Е. Курманов^{1,2*}, Ұ. Нұрланбек², Г. Рабиғұлова^{1,2},
М. Әділ², Б. Бекмұрат², Г. Түзен², А. Уразалина^{1,2,3}

¹Ашық түрдегі ұлттық нанотехнологиялық зертхана, Алматы, Қазақстан

²Әл-Фараби атындағы Қазақ ұлттық университеті, Алматы, Қазақстан

³Ядролық физика институты, Алматы, Қазақстан

*e-mail: kurmanov.yergali@kaznu.kz

Сен кеңістік-уақытындағы шағын нысандар айналасындағы квазипериодтық тербелістер

Бұл мақалада релятивистік прецессия моделі шеңберінде сегіз нейтрон жұлдызы бар төмен массалы рентгендік қос жұлдыздық жүйелерде байқалған килогерцтік (кГц) квазипериодтық тербелістерді (КПТ) зерттелген. Дөңгелек орбиталармен қозғалатын сынақ бөлшектерінің негізгі (эпициклдік) жиіліктері статикалық Сен кеңістік-уақытында есептелді. Кеплерлік және эпициклдік жиіліктерді бақылаудан алынған төменгі және жоғарғы КПТ жиілік жұптарымен (f_L, f_U) сәйкестендіру арқылы шағын нысандардың массалары мен электр зарядтарын анықталды. Сонымен қатар, Акайке және Байес ақпараттық критерийлерін (AIC/BIC) қолдана отырып Шварцшильд кеңістік-уақытындағы нәтижелермен салыстырылды. Шварцшильд геометриясы төрт дереккөз үшін физикалық тұрғыдан үйлесімді нәтиже көрсетті. Ал GX 5–1 және GX 340+0 үшін Сен метрикасы Шварцшильд геометриясымен жуық болатыны анықталды, яғни бұл шағын нысандар үшін электр зарядын енгізудің қажет еместігін білдіреді. Қалған төрт дереккөз үшін Сен метрикасы статистикалық тұрғыдан жақсырақ сәйкестік бергенімен, алынған үлкен массалар мен үлкен электр зарядтары нейтрон жұлдыздарының физикасына сәйкес келмейді. Осылайша, біз

статикалық Сен метрикасы нейтрон жұлдыз жүйелеріндегі кГц КПО-ді физикалық тұрғыдан дұрыс сипаттай алмайды деген қорытындыға келеміз.

Түйін сөздер: аккрециялық дискілер, квазипериодтық тербелістер, қара құрдымдар, нейтрон жұлдыздары, рентгендік қос жұлдыздық жүйелер, релятивистік прецессия моделі.

Т. Конысбаев^{1,2}, Е. Курманов^{1,2*}, У. Нурланбек², Г. Рабиғулова^{1,2},
М. Адил², Б. Бекмурат², Г. Тузен², А. Уразалина^{1,2,3}

¹Национальная нанотехнологическая лаборатория открытого типа, Алматы, Казахстан

²Казахский национальный университет имени аль-Фараби, Алматы, Казахстан

³Институт ядерной физики, Алматы, Казахстан

*e-mail: kurmanov.yergali@kaznu.kz

Квазипериодические осцилляции вокруг компактных объектов в пространстве-времени Сена

В данной работе исследованы килогерцовые (кГц) квазипериодические осцилляции (КПО), наблюдаемые в восьми рентгеновских двойных системах с нейтронными звёздами низкой массы, в рамках модели релятивистской прецессии. Основные (эпициклические) частоты тестовых частиц на круговых орбитах вычисляются в статическом пространстве–времени Сена. Путём подгонки кеплеровских и эпициклических частот к наблюдаемым парам низких и высоких частот КПО (f_L, f_U) мы определяем массы и электрические заряды компактных объектов и сравниваем результаты с полученными в пространстве–времени Шварцшильда с использованием информационных критериев Акаике и Байеса (AIC/BIC). Мы находим, что геометрия Шварцшильда даёт физически согласованные аппроксимации для четырёх источников, тогда как для GX 5–1 и GX 340+0 пространство–время Сена становится практически неотличимым от Шварцшильдовского, что указывает на отсутствие необходимости вводить электрический заряд. Хотя метрика Сена обеспечивает статистически более точные подгонки для оставшихся четырёх источников, полученные большие массы и большие электрические заряды несовместимы с физикой нейтронных звёзд. Следовательно, мы заключаем, что статическая метрика Сена не даёт физически реалистичного описания кГц КПО в системах с нейтронными звёздами.

Ключевые слова: аккреционные диски, квазипериодические осцилляции, чёрные дыры, нейтронные звёзды, рентгеновские двойные системы, модель релятивистской прецессии.

Introduction

Rotating neutron stars, commonly observed as pulsars, can exist either as isolated objects or as members of binary systems. In binaries, accretion from a companion star can significantly increase both the mass and the spin of the neutron star, as angular momentum is transferred through the accretion flow. As a result, pulsars in LMXBs can reach spin frequencies of several hundred hertz, with the fastest known pulsar rotating at 716 Hz [1]. These rapidly rotating neutron stars provide natural laboratories for studying gravity and dense-matter physics in the strong-field regime.

Quasi-periodic oscillations (QPOs) appear as narrow peaks in the Fourier power spectra of X-ray light curves and were first identified in cataclysmic variables [2]. They have since been widely observed in neutron-star and black-hole LMXBs [3,4], as well

as in ultraluminous X-ray sources and active galactic nuclei (e.g. [5-8]). In many systems, the measured QPO frequencies indicate that the oscillations originate in the innermost regions of the accretion flow, close to the compact object, making QPOs sensitive probes of strong-gravity effects [9]. However, their diagnostic potential remains limited by the absence of a universally accepted physical model of their origin.

In neutron-star LMXBs, QPOs are commonly classified into low- and high-frequency types. Low-frequency QPOs typically span the range from about 0.1 to 60 Hz and, in Z sources, are traditionally divided into horizontal, normal and flaring-branch oscillations [10-12]. Although Atoll sources do not follow the same colour–colour diagram tracks [13], they exhibit analogous low-frequency QPO features

[14]. By contrast, High-frequency QPOs lie in the 300-1000 Hz range and are therefore known as kilohertz (kHz) QPOs [3]. These often appear as a pair of peaks, referred to as the upper and lower kHz QPOs, whose frequencies typically increase with X-ray flux. Several neutron-star LMXBs display such twin kHz QPOs, which are believed to be generated in the innermost regions of the accretion disk [9]. In this work, we focus on these systems and use their QPO properties to test gravity models in the strong-field regime.

We deliberately adopt the static Sen spacetime in order to isolate the physical effect of the string-inspired electric charge on the QPO phenomenology [15]. Although the rotating Sen solution contains both spin and charge, the simultaneous presence of these two parameters introduces a strong degeneracy in QPO modeling [16]. By setting the spin to zero, we show that the Sen charge alone can reproduce the

Sen spacetime

The geometry of the Sen spacetime is given by the metric of the form [21]:

$$ds^2 = -N(r)dt^2 + \frac{1}{N(r)}dr^2 + r^2 \left(1 + \frac{2b}{r}\right) d\theta^2 + \left(1 + \frac{2b}{r}\right) \sin^2 \theta d\varphi^2, \quad (1)$$

$$N(r) = \left[1 - \frac{2(M-b)}{r}\right] \left(1 + \frac{2b}{r}\right)^{-1},$$

here $b = \frac{\tilde{Q}^2}{2M}$, where \tilde{Q} denotes the electric charge of the compact object (in what follows, we set $\tilde{Q} = \frac{Q}{2}$), M represents its gravitational mass. The radius of the event horizon is obtained from the condition $N(r) = 0$, which yields

$$r_h = 2(M - b). \quad (2)$$

The t and φ components of test particle's 4-velocity may be expressed

$$u^t = \frac{Eg_{\varphi\varphi} + Lg_{t\varphi}}{g_{t\varphi}^2 - g_{tt}g_{\varphi\varphi}}, \quad u^\varphi = \frac{Eg_{t\varphi} + Lg_{tt}}{g_{t\varphi}^2 - g_{tt}g_{\varphi\varphi}}. \quad (3)$$

Employing the normalization condition of the four-velocity, $u^\mu u_\mu = -1$, one obtains the following:

$$g_{rr}\dot{r}^2 + g_{\theta\theta}\dot{\theta}^2 = V_{eff}(r, \theta, E, L), \quad (4)$$

observed QPO frequencies and mimic the phenomenology of a rotating Kerr black hole. This strategy allows us to directly assess whether QPO observations can discriminate between classical rotation and string-theoretic corrections to gravity.

This work examines the circular and epicyclic motion of test particles around a compact object and evaluates the fundamental frequencies predicted by several metric models. We also analyze the QPOs observed in eight neutron-star sources: Cir X-1, GX 5-1, GX 17+2, GX 340+0, Sco X-1, 4U1608-52, 4U1728-34, and 4U0614+091 [17-20]. Our primary goal is to investigate the Sen metric, charged solution arising in the low-energy limit of heterotic string theory. Throughout this paper we adopt the $(-, +, +, +)$ signature and use geometric units ($G = c = 1$), providing astrophysically relevant expressions also in physical units.

where $\dot{r} = u^r = \frac{dr}{d\lambda}$, $\dot{\theta} = u^\theta = \frac{d\theta}{d\lambda}$, the parameter λ is the affine parameter along the geodesic, corresponding to the proper time for a massive test particle. The effective potential V_{eff} can then be written as

$$V_{eff} = \frac{E^2 g_{\varphi\varphi} + 2ELg_{t\varphi} + L^2 g_{tt}}{g_{t\varphi}^2 - g_{tt}g_{\varphi\varphi}} - 1. \quad (5)$$

From these relations, one can obtain the Keplerian angular velocity of massive test particles-equal to the azimuthal orbital frequency-as well as the corresponding general expressions for the specific energy and the orbital angular momentum per unit mass for particles moving on circular trajectories.

$$\Omega_\varphi = \frac{-\partial_r g_{t\varphi} \pm \sqrt{(\partial_r g_{t\varphi})^2 - (\partial_r g_{tt})(\partial_r g_{\varphi\varphi})}}{\partial_r g_{\varphi\varphi}}, \quad (6)$$

$$E = -\frac{-g_{tt} + g_{t\varphi}\Omega_\varphi}{\sqrt{-g_{tt} - 2g_{t\varphi}\Omega_\varphi - g_{\varphi\varphi}\Omega_\varphi^2}}, \quad (7)$$

$$L = \frac{g_{t\varphi} + g_{\varphi\varphi}\Omega_\varphi}{\sqrt{-g_{tt} - 2g_{t\varphi}\Omega_\varphi - g_{\varphi\varphi}\Omega_\varphi^2}}, \quad (8)$$

where “+” indicates co-rotating(prograde) orbits and “-” denotes counter-rotating (retrograde) orbits. Because the metric is static ($g_{t\varphi} = 0$), all expressions

for Ω_ϕ , E , and L take their simplified forms obtained by setting $g_{t\phi} = 0$ in the general axisymmetric formulas.

The frequencies of epicyclic motion

The radial and vertical frequencies can be calculated by the following harmonic oscillator equations [22]

$$\frac{d^2 \delta r}{dt^2} + \Omega_r^2 \delta r = 0, \quad \frac{d^2 \delta \theta}{dt^2} + \Omega_\theta^2 \delta \theta = 0, \quad (9)$$

here

$$\Omega_r^2 = -\frac{1}{2g_{rr}(u^t)^2} \frac{\partial^2 V_{eff}}{\partial r^2}, \quad (10)$$

$$\Omega_\theta^2 = -\frac{1}{2g_{\theta\theta}(u^t)^2} \frac{\partial^2 V_{eff}}{\partial \theta^2}. \quad (11)$$

Using the Sen metric line element given in Eq. (1), together with the general expressions for the conserved energy and angular momentum in Eqs. (6)-(8), the specific energy E , the specific angular momentum L , and the temporal component of the four-velocity u^t for massive particles on circular equatorial orbits can be written in explicit form as

$$E = \frac{Q^2 + 4M(r - 2M)}{\sqrt{Q^2 + 4Mr}} \times \frac{\sqrt{Q^2 + 8Mr}}{\sqrt{F}}, \quad (12)$$

$$L = \frac{2\sqrt{2Mr}\sqrt{Q^2 + 4Mr}}{\sqrt{F}}, \quad (13)$$

Data analysis and results

To estimate the model parameters, we apply a nonlinear least-squares fitting procedure implemented through the NonlinearModelFit routine of the Wolfram Language (Mathematica). This method, based on a Levenberg–Marquardt–type algorithm, is well suited for nonlinear dependencies between observables and model parameters. Similar fitting techniques have been widely used in astrophysical studies, including pulsar spectral modelling [24], broadband Faraday-rotation analyses of AGN [25], radio-background calibration [26], and cosmological parameter estimation [27]. The routine returns the best-fitting parameter values along with statistical uncertainties and goodness-of-fit

$$u^t = \frac{\sqrt{(Q^2 + 4Mr)(Q^2 + 8Mr)}}{\sqrt{F}}. \quad (14)$$

The corresponding Keplerian and vertical epicyclic frequencies are identical in the non-rotating Sen spacetime and are given by

$$\Omega_\phi^2 = \Omega_\theta^2 = \frac{128M^4}{(Q^2 + 4Mr)^2(Q^2 + 8Mr)}. \quad (15)$$

The radial epicyclic frequency, governing the stability of circular motion against radial perturbations, takes the form

$$\Omega_r^2 = \frac{\Omega_\phi^2 [Q^2(F + 16M^2r^2) + 64M^3r^2(r - 6M)]}{4Mr(Q^2 + 4Mr)^2}, \quad (16)$$

where the auxiliary function F is defined as

$$F = Q^4 - 96M^3r + 12Q^2Mr - 8M^2(Q^2 - 4r^2), \quad (17)$$

and the Keplerian, radial epicyclic, and vertical oscillation frequencies are defined as

$$f_\phi = \frac{\Omega_\phi}{2\pi}, \quad f_r = \frac{\Omega_r}{2\pi}, \quad f_\theta = \frac{\Omega_\theta}{2\pi}. \quad (18)$$

Within the relativistic precession model (RPM), the lower QPO frequency f_L is identified with the periastron–precession frequency [23]

$$f_L = f_U - f_r, \quad (19)$$

while the upper QPO frequency f_U corresponds to the Keplerian frequency, $f_U = f_\phi$.

diagnostics, enabling a reliable assessment of the physical plausibility of the model.

To constrain the parameters of the Sen spacetime, we perform a simultaneous nonlinear least-squares fit of the model-predicted upper and lower kHz QPO frequencies to the observed data. For each observation k we compare the measured pair $(f_{U,obs,k}, f_{L,obs,k})$ with the corresponding theoretical predictions $f_U(r_k, \theta)$ and $f_L(r_k, \theta)$ derived from the orbital and radial epicyclic frequencies of the Sen metric. The best-fitting parameter set θ ($\theta = M, Q$) is obtained by minimising the χ^2 function in Eq. (20) using a nonlinear least-squares routine based on the

Mathematica function `NonlinearModelFit`. The procedure yields the optimal values of the spacetime parameters together with their formal uncertainties and the reduced chi-square, which we use to assess the quality of the fit and the viability of the QPO model.

$$\chi^2 = \sum_{k=1}^N \left[\frac{f_{U,k}^{obs} - f_{U,k}}{\sigma_{U,k}} \right]^2, \quad (20)$$

$\sigma_{U,k}$ denote the statistical uncertainties of the upper and lower kHz QPO frequencies at the k -th observation.

To further assess the quality of the fits and to compare different QPO models, we compute two commonly used statistical criteria: the Akaike Information Criterion (*AIC*) and the Bayesian Information Criterion (*BIC*). Both criteria penalize models with a larger number of free parameters and therefore provide a quantitative tool for model selection.

The *AIC* is defined as

$$AIC = \chi^2 + 2i, \quad (21)$$

where χ^2 is the minimum chi-square value obtained from the fit and i is the number of free model parameters.

The *BIC* introduces a stronger penalty for model complexity and is given by

$$BIC = \chi^2 + i \ln N, \quad (22)$$

where N is the total number of observational data points used in the fitting (i.e., the number of QPO frequencies included in the analysis).

Lower values of *AIC* and *BIC* indicate a more favourable model. These criteria enable us to evaluate whether the improvement in χ^2 justifies the introduction of additional parameters and therefore provide an objective basis for comparing alternative QPO models in the Sen spacetime.

When comparing different models, the strength of the evidence against a given model - or, equivalently, in favour of the reference model - can be evaluated through the *BIC* and the *AIC* difference. In particular,

- ΔAIC and $\Delta BIC \in [0,3]$ indicates weak evidence,
- ΔAIC and $\Delta BIC \in [3,6]$ indicates mild evidence,
- ΔAIC and $\Delta BIC > 6$ indicates strong evidence.

Accordingly, the preferred model is the one with the lowest *AIC*(*BIC*) value, although each case must be examined individually. By definition, the reference model has ΔAIC (*BIC*) = 0.

Table 1 – Best-fit parameters for the Schwarzschild and the Sen spacetimes.

Source	Metric	$M(M_{\odot})$	Q	<i>AIC</i>	<i>BIC</i>	ΔAIC	ΔBIC
Cir X1	Schwarzschild	2.04	–	103	102	0	0
	Sen	5.22	21.57	105	104	2	2
GX 5-1	Schwarzschild	2.16	–	213	211	0	0
	Sen	2.16	$-1.3 \cdot 10^{-5}$	216	213	3	3
GX 17+2	Schwarzschild	2.31	–	126	125	0	0
	Sen	2.71	5.45	129	127	3	3
GX 340+0	Schwarzschild	2.10	–	124	123	0	0
	Sen	2.10	$3.11 \cdot 10^{-6}$	126	125	2	2
Sco X1	Schwarzschild	1.96	–	476	472	209	215
	Sen	7.09	29.14	267	257	0	0
4U1608–52	Schwarzschild	1.96	–	143	141	27	27
	Sen	6.76	27.67	116	114	0	0
4U1728–34	Schwarzschild	1.71	–	86	86	10	10
	Sen	6.64	27.46	76	76	0	0
4U0614+091	Schwarzschild	1.90	–	252	250	48	50
	Sen	8.23	34.28	204	200	0	0

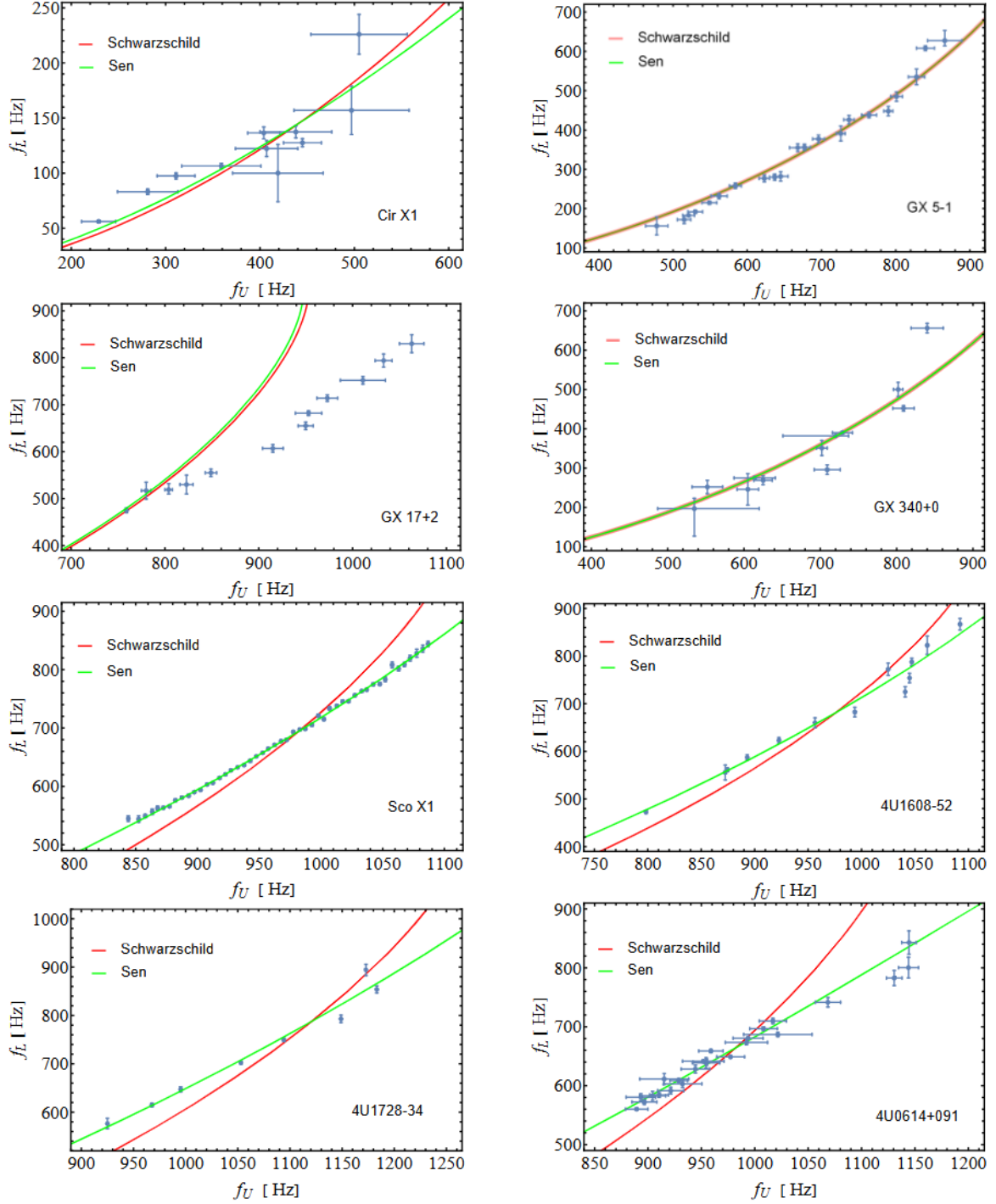


Figure 1 – Plots of f_L vs f_U frequencies of the QPO data sets considered in this work (dark cyan data with error bars).

Discussion

In this work we have applied the relativistic precession model in the non-rotating Sen spacetime and compared its predictions with those of the Schwarzschild metric for the same set of kHz QPO data from eight LMXB sources. The best-fit parameters summarized in Table 1 allow us to discuss both the statistical performance of the Sen model and the physical plausibility of the inferred masses and

charges. Using the best-fit parameters listed in Table 1, the quality of the resulting frequency relations is illustrated in Fig. 1, where the observed (f_L, f_U) data for all eight sources are compared with the predictions of the Schwarzschild and Sen spacetimes.

For the sources Cir X-1, GX 5-1, GX 17+2 and GX 340+0 the Schwarzschild metric provides the lowest AIC and BIC values, while the Sen spacetime

yields only a marginal deterioration of the fit quality (with $\Delta AIC, \Delta BIC \leq 3$). In two cases, GX~5-1 and GX~340+0, the best-fit charge is extremely small and compatible with zero within the numerical accuracy of our analysis, so that the Sen solution effectively reduces to the Schwarzschild one. For Cir~X-1 and GX~17+2 the Sen model prefers higher masses and non-vanishing charge, but the improvement with respect to the neutral case is not statistically significant according to standard information-criterion thresholds. These results indicate that, for these four systems, the available QPO data do not require the presence of a non-zero electric charge: Schwarzschild geometry already reproduces the observed frequency pairs at a level comparable to the charged Sen spacetime.

A markedly different behavior is found for Sco X-1, 4U~1608--52, 4U~1728--34 and 4U~0614+091. For these sources the fits in the Sen metric lead to substantially lower AIC and BIC values than in the Schwarzschild case, with ΔAIC and ΔBIC well above 6. In the usual interpretation of these criteria, such differences correspond to strong evidence in favour of the Sen spacetime. Nevertheless, the resulting fit requires masses in the range $M \simeq 6 - 8M_{\odot}$ together with substantial charge values Q . If the compact objects in these systems are indeed neutron stars, as suggested by independent observational arguments (burst properties, spectral states, and, in some cases, spin measurements), such large masses are difficult to reconcile with standard equations of state and with the commonly accepted upper limit for neutron star masses of about $3.2M_{\odot}$ (see, e.g., [28]). In this sense, the Sen fits for Sco X-1, 4U 1608-52, 4U 1728-34 and 4U 0614+091 are statistically very good, but they tend to push the system parameters into a regime that is more typical of stellar-mass black holes than of neutron stars.

This tension between statistical preference and physical consistency is also apparent when our findings are confronted with previous analyses based on different space times. In the Hartle–Thorne study of the same QPO sample [29], it was shown that including spin and quadrupole moments can slightly improve the description of some sources, but only a subset of the best-fit configurations remains compatible with realistic neutron star parameters once priors on the dimensionless spin j are imposed. For instance, restricting j to the range expected for slowly rotating neutron stars leads to acceptable solutions mainly for a few sources, while allowing very large j often produces unphysical masses. In that analysis the Schwarzschild metric already provides

robust and stable mass estimates for most systems, whereas the extension to Hartle–Thorne geometries mainly highlights the degeneracy between spin and quadrupole parameters and the limited constraining power of current QPO data. Our analysis in the Sen spacetime reveals an analogous pattern: the introduction of an additional parameter (the charge) can significantly lower the AIC/BIC values for some sources, but the corresponding masses and charges are often too extreme to be comfortably identified with a canonical neutron star.

It is also instructive to compare our findings with the original formulation of the relativistic precession model for Sco X-1 and 4U 1608-52 [23]. In that work, the observed pairs of kHz QPOs were successfully interpreted as the azimuthal and periastron precession frequencies of test particles orbiting close to the innermost stable circular orbit in a Schwarzschild (or slowly rotating) spacetime, yielding neutron star masses of the order of $\sim 2M_{\odot}$ and demonstrating that the QPOs probe the strong-field regime of general relativity. In contrast, when the same sources are reanalyzed in the Sen spacetime, we obtain a significantly better statistical fit at the price of inferring masses exceeding $6M_{\odot}$. This suggests that if Sco X-1 and 4U 1608-52 indeed host neutron stars, either the simple identification of the kHz QPOs with geodesic frequencies must be revised, or the Sen metric is not the appropriate effective description of their exterior gravitational field.

Overall, our study points to a twofold conclusion. On the one hand, the QPO data for several LMXBs can be reproduced extremely well in the Sen spacetime, and information criteria clearly favour this geometry over the Schwarzschild one for four sources. On the other hand, the corresponding best-fit parameters are often difficult to reconcile with the neutron star interpretation, in agreement with previous indications that current QPO data alone are insufficient to uniquely determine both the space-time metric and the stellar parameters [19, 23, 28, 29]. Future work should therefore include (i) a joint treatment of rotation and charge in the full Kerr-Sen spacetime, (ii) the incorporation of independent mass and radius constraints from spectroscopy and pulse profile modelling, and (iii) possible extensions of the RPM that account for non-geodesic effects in thick or magnetized accretion flows. Only by combining these ingredients will it be possible to assess whether QPOs can provide robust constraints on the parameters of Sen black holes or whether the charged dilatonic extension of general relativity remains disfavoured in realistic neutron star systems.

Conclusions

In this work, we have investigated the applicability of the relativistic precession model to the non-rotating Sen spacetime and tested its ability to reproduce the observed kHz QPOs from eight neutron-star LMXBs. By fitting the theoretical expressions for the Keplerian, radial epicyclic and vertical frequencies to the measured pairs f_L, f_U , we inferred the corresponding masses and electric charges of the compact objects and compared the statistical performance of the Sen metric with that of the Schwarzschild geometry.

Our analysis leads to several key conclusions. First, for four sources (Sco X-1, 4U 1608-52, 4U 1728-34 and 4U 0614+091), the Sen spacetime provides a substantially better statistical description of the QPO data, yielding markedly lower AIC/BIC values than Schwarzschild. However, corresponding best-fit masses, $M \simeq 6 - 8M_\odot$, together with large charge values, are incompatible with a neutron-star interpretation. Thus, although the Sen geometry succeeds statistically, it fails to provide physically acceptable stellar parameters for these systems. Second, for the remaining four sources (Cir X-1, GX 5-1, GX 17+2 and GX 340+0), the improvement produced by the Sen metric is negligible or absent. In two of these systems (GX 5-1 and GX 340+0), the best-fit charge is essentially zero, indicating that the Sen solution naturally reduces to the Schwarzschild limit. For Cir X-1 and GX 17+2 the Sen model introduces non-zero charge and larger masses, but without a statistically significant gain. Therefore, for

these sources the Schwarzschild geometry remains fully adequate.

Taken together, these results show that although the Sen spacetime provides additional flexibility in adjusting the QPO frequency relation, this flexibility generally drives the inferred stellar parameters beyond the allowed range for neutron stars. When the Sen model fits extremely well, it does so at the expense of physical plausibility. This trend is consistent with previous studies based on other deformed or extended metrics, which similarly find that geodesic QPO models can formally fit the data but do not yield a universal and physically meaningful description across all LMXBs.

Overall, our findings indicate that the non-rotating Sen spacetime does not constitute a viable alternative to the Schwarzschild geometry for modelling kHz QPOs in neutron-star systems. Future work should extend the present analysis to the full Kerr-Sen metric, incorporate independent mass-radius constraints, and explore non-geodesic contributions to QPO frequencies. Such developments will be essential for assessing whether charged dilatonic spacetimes can offer a consistent interpretation of high-frequency variability in accreting compact objects.

Acknowledgments

TK acknowledges grant No. AP32722555, YeK acknowledges grant No. AP23488743 from the Science Committee of the Ministry of Science and Higher Education of the Republic of Kazakhstan.

Author Contributions:

T. Konysbayev: Supervision, Conceptualization, Writing; **Y. Kurmanov:** Investigation, Writing – review & editing, Data curation; **U. Nurlanbek:** Software, Methodology, Validation; **G. Rabigulova:** Software, Investigation, Formal Analysis; **M. Adil:** Investigation, Software, Writing; **B. Bekmurat:** Investigation, Software, Writing; **G. Tuzen:** Investigation, Software, Writing; **A. Urazalina:** Formal Analysis, Methodology, Validation.

References

1. J.W.T. Hessels, S. M. Ransom, I.H. Stairs, P.C.C. Freire, V.M. Kaspi and F. Camilo, A Radio Pulsar Spinning at 716 Hz, *Science* **311**, 1901–1904 (2006). <https://doi.org/10.1126/science.1123430>
2. J. Patterson, E. L. Robinson and R. E. Nather, Rapid and ultrarapid oscillations in RU Pegasi, *Astrophysical Journal* **214**, 144–151 (1977). <https://doi.org/10.1086/155239>
3. M. van der Klis, Millisecond Oscillations in X-Ray Binaries, *Annual Review of Astronomy and Astrophysics* **38**, 717–760 (2000). <https://doi.org/10.1146/annurev.astro.38.1.717>
4. R. A. Remillard and J. E. McClintock, X-Ray Properties of Black-Hole Binaries, *Annual Review of Astronomy and Astrophysics* **44**, 49–92 (2006). <https://doi.org/10.1146/annurev.astro.44.051905.092532>
5. T.E. Strohmayer and R.F. Mushotzky, Discovery of x-ray quasi-periodic oscillations from an ultraluminous x-ray source in m82: evidence against beaming, *Astrophysical Journal Letters* **586**, L61–L64 (2003). <https://doi.org/10.1086/374732>
6. M. Bachetti et al., An ultraluminous X-ray source powered by an accreting neutron star, *Nature* **514**, 202–204 (2014). <https://doi.org/10.1038/nature13791>
7. M. Gierliński, M. Middleton, M. Ward and C. Done, A periodicity of ~ 1 hour in X-ray emission from the active galaxy RE J1034+396, *Nature* **455**, 369–371 (2008). <https://doi.org/10.1038/nature07277>

8. M. Middleton and C. Done, The X-ray binary analogy to the first AGN quasi-periodic oscillation, *Monthly Notices of the Royal Astronomical Society* **403**, 9–16 (2010). <https://doi.org/10.1111/j.1365-2966.2009.15969.x>
9. W. Lewin and M. van der Klis, *Accreting Neutron Stars in Low-Mass X-Ray Binary Systems*, (Cambridge University Press, 2006).
10. G. Hasinger and M. van der Klis, Two patterns of correlated X-ray timing and spectral behaviour in low-mass X-ray binaries, *Astronomy and Astrophysics* **225**, 79–96 (1989). <https://ui.adsabs.harvard.edu/abs/1989A&A...225...79H>
11. J. Middleditch and W. C. Priedhorsky, Discovery of rapid quasi-periodic oscillations in Scorpius X-1, *Astrophysical Journal* **306**, 230–236 (1986). <https://doi.org/10.1086/164335>
12. M. van der Klis et al., Intensity-dependent quasi-periodic oscillations in the X-ray flux of GX5-1, *Nature* **316**, 225–230 (1985). <http://dx.doi.org/10.1038/316225a0>
13. M. van der Klis, Quasi-Periodic Oscillations and Noise in Low-Mass X-Ray Binaries, *Annual Review of Astronomy and Astrophysics* **27**, 517–553 (1989). <https://doi.org/10.1146/annurev.aa.27.090189.002505>
14. S.E. Motta et al., Links between quasi-periodic oscillations and accretion states in neutron star low-mass X-ray binaries, *Monthly Notices of the Royal Astronomical Society* **468**, 2311–2324 (2017). <https://doi.org/10.1093/mnras/stx570>
15. K. Boshkayev et al., Quasiperiodic oscillations for spherically symmetric regular black holes, *Physical Review D* **108**, 044063 (2023). <https://doi.org/10.1103/PhysRevD.108.044063>
16. K. Boshkayev, O. Luongo and M. Muccino, Numerical analysis of quasiperiodic oscillations with spherical spacetimes, *Physical Review D* **108**, 124034 (2023). <https://doi.org/10.1103/PhysRevD.108.124034>
17. K. Boshkayev et al., Quasi-periodic oscillations in rotating and deformed space-times, *Monthly Notices of the Royal Astronomical Society* **531**, 3876–3887 (2024). <https://doi.org/10.1093/mnras/stae1388>
18. K. Boshkayev et al., What can we extract from quasiperiodic oscillations? *Gravitation and Cosmology* **20**, 233–239 (2014). <https://doi.org/10.1134/S0202289314040033>
19. K. Boshkayev and M. Muccino, Constraints on the Sen black hole mass and charge from quasi-periodic oscillations, *European Physical Journal C* **85**, 1-8 (2025). <https://doi.org/10.1140/epjc/s10052-025-15167-w>
20. W. Kluźniak and M.A. Abramowicz, Resonant Oscillations of Accretion Flow and Khz QPOS, *Astrophysics and Space Science* **300**, 143–148 (2005). <https://doi.org/10.1007/s10509-005-0020-6>
21. L. Stella and M. Vietri, kHz Quasiperiodic Oscillations in Low-Mass X-Ray Binaries as Probes of General Relativity in the Strong-Field Regime, *Physical Review Letters* **82**, 17–20 (1999). <https://doi.org/10.1103/PhysRevLett.82.17>
22. H. Ardavan et al., Radio spectra of pulsars fitted with the spectral distribution function of the emission from their current sheet, *Monthly Notices of the Royal Astronomical Society* **529**, 3744–3750 (2024). <https://doi.org/10.1093/mnras/stae774>
23. S.P. O’Sullivan et al., Complex Faraday depth structure of active galactic nuclei as revealed by broad-band radio polarimetry, *Monthly Notices of the Royal Astronomical Society* **421**, 3300–3315 (2012). <https://doi.org/10.1111/j.1365-2966.2012.20554.x>
24. T.J. Mozdzen et al., Spectral index of the diffuse radio background between 50 and 100MHz, *Monthly Notices of the Royal Astronomical Society* **483**, 4411–4425 (2019). <https://doi.org/10.1093/mnras/sty3410>
25. A. Mohammadi et al., A possible role for the merger of clusters/voids in the cosmological expansion, *Monthly Notices of the Royal Astronomical Society* **525**, 3274–3280 (2023). <https://doi.org/10.1093/mnras/stad2514>
26. G. Srinivasan, The maximum mass of neutron stars, *The Astronomy and Astrophysics Review* **11**, 67–96 (2002). <https://doi.org/10.1007/s001590200016>
27. K. Boshkayev et al., Numerical analysis of quasiperiodic oscillations in the Hartle-Thorne spacetime (2025). <https://doi.org/10.48550/arXiv.2506.11581>
28. S. Boutloukos et al., Discovery of Twin kHz QPOs in the Peculiar X-Ray Binary Circinus X-1, *The Astrophysical Journal* **653**, 1435–1444 (2006). <http://dx.doi.org/10.1086/508934>
29. R. Wijnands et al., Discovery of Kilohertz Quasi-periodic Oscillations in the Z Source GX 5–1, *The Astrophysical Journal Letters* **504**, L35–L38 (1998). <https://doi.org/10.1086/311564>
30. P.G. Jonker et al., Low- and high-frequency variability as a function of spectral properties in the bright X-ray binary GX 5–1, *Monthly Notices of the Royal Astronomical Society* **333**, 665–678 (2002). <https://doi.org/10.1046/j.1365-8711.2002.05442.x>
31. J. Homan et al., RXTE Observations of the Neutron Star Low-Mass X-Ray Binary GX 17+2: Correlated X-Ray Spectral and Timing Behavior, *The Astrophysical Journal* **568**, 878–900 (2002). <https://doi.org/10.1086/339057>
32. P.G. Jonker et al., The Power Spectral Properties of the Z Source GX 340+0, *The Astrophysical Journal* **537**, 374–386 (2000). <http://dx.doi.org/10.1086/309029>
33. M. Méndez and M. van der Klis, The harmonic and sideband structure of the kilohertz quasi-periodic oscillations in Sco X-1, *Monthly Notices of the Royal Astronomical Society* **318**, 938–942 (2000). <https://doi.org/10.1046/j.1365-8711.2000.03788.x>
34. M. Méndez et al., Kilohertz Quasi-periodic Oscillation Peak Separation Is Not Constant in the Atoll Source 4U 1608–52, *The Astrophysical Journal* **505**, L23–L26 (1998). <https://doi.org/10.1086/311600>
35. M. Méndez and M. van der Klis, Precise Measurements of the Kilohertz Quasi-periodic Oscillations in 4U 1728–34, *The Astrophysical Journal* **517**, L51–L54 (1999). <https://doi.org/10.1086/312025>

36. E.C. Ford et al., Energy Spectra and High-Frequency Oscillations in 4U 0614+091, *The Astrophysical Journal* **486**, L47–L50 (1997). <https://doi.org/10.1086/310827>

Article history: received: 12 January 2026; accepted: 25 February 2026.

Мақала тарихы: түсті: 12 қаңтар 2026; қабылданды: 25 ақпан 2026.

История статьи: получено: 12 января 2026; принято: 25 февраля 2026.

Information about authors:

Talgar Konysbayev – PhD, Leading Researcher, National Nanotechnology Laboratory of Open Type, Al-Farabi Kazakh National University, Almaty, Kazakhstan, e-mail: talgar_777@mail.ru

Yergali Kurmanov – PhD, Associate Professor, Leading Researcher, National Nanotechnology Laboratory of Open Type, Al-Farabi Kazakh National University, Almaty, Kazakhstan, e-mail: kurmanov.yergali@kaznu.kz

Ulpan Nurlanbek – 1st-year PhD student in Physics and Astronomy, Al-Farabi Kazakh National University, Almaty, Kazakhstan, e-mail: ulpan.nurlanbek02@gmail.com

Guldana Rabigulova – 1st-year PhD student in Physics and Astronomy, Al-Farabi Kazakh National University, Almaty, Kazakhstan, e-mail: guldanaberikhanovna@gmail.com

Makhabbat Adil – 1st-year Master’s student in Physics and Astronomy, Al-Farabi Kazakh National University, Almaty, Kazakhstan, e-mail: adilmahabbat@gmail.com

Bagzhan Bekmurat – 1st-year Master’s student in Physics and Astronomy, Al-Farabi Kazakh National University, Almaty, Kazakhstan, e-mail: bagzhan159@gmail.com

Galiya Tuzen – 1st-year Master’s student in Physics and Astronomy, Al-Farabi Kazakh National University, Almaty, Kazakhstan, e-mail: galiyatuzenn@gmail.com

Ainur Urazalina – PhD, Associate Professor, Leading Researcher, National Nanotechnology Laboratory of Open Type, Al-Farabi Kazakh National University, Almaty, Kazakhstan, e-mail: y.a.a.707@mail.ru

Авторлар туралы мәлімет:

Талғар Қонысбаев – PhD, ЖФҚ, Ашық түрдегі ұлттық нанотехнологиялық зертхана, әл-Фараби атындағы Қазақ ұлттық университеті, Алматы, Қазақстан, e-mail: talgar_777@mail.ru

Ергали Курманов – PhD, қауымдастырылған профессор, ЖФҚ, Ашық түрдегі ұлттық нанотехнологиялық зертхана, әл-Фараби атындағы Қазақ ұлттық университеті, Алматы, Қазақстан, e-mail: kurmanov.yergali@kaznu.kz

Ұлпан Нұрланбек – физика және астрономия мамандығының 1 курс докторанты, әл-Фараби атындағы Қазақ ұлттық университеті, Алматы, Қазақстан, e-mail: ulpan.nurlanbek02@gmail.com

Гүлдана Рабиғұлова – физика және астрономия мамандығының 1 курс докторанты, әл-Фараби атындағы Қазақ ұлттық университеті, Алматы, Қазақстан, e-mail: guldanaberikhanovna@gmail.com

Махаббат Әділ – физика және астрономия мамандығының 1 курс магистранты, әл-Фараби атындағы Қазақ ұлттық университеті, Алматы, Қазақстан, e-mail: adilmahabbat@gmail.com

Бағжан Бекмұрат – физика және астрономия мамандығының 1 курс магистранты, әл-Фараби атындағы Қазақ ұлттық университеті, Алматы, Қазақстан, e-mail: bagzhan159@gmail.com

Галия Тузен – физика және астрономия мамандығының 1 курс магистранты, әл-Фараби атындағы Қазақ ұлттық университеті, Алматы, Қазақстан, e-mail: galiyatuzenn@gmail.com

Айнур Уразалина – PhD, қауымдастырылған профессор, ЖФҚ, Ашық түрдегі ұлттық нанотехнологиялық зертхана, әл-Фараби атындағы Қазақ ұлттық университеті, Алматы, Қазақстан, e-mail: y.a.a.707@mail.ru

Информация об авторах:

Конисбаев Талғар – PhD, научный сотрудник, Национальная открытая лаборатория нанотехнологий, Казахский национальный университет им. Аль-Фараби, Алматы, Казахстан, e-mail: talgar_777@mail.ru

Курманов Ергали – PhD, доцент, научный сотрудник, Национальная открытая лаборатория нанотехнологий, Казахский национальный университет им. Аль-Фараби, Алматы, Казахстан, e-mail: kurmanov.yergali@kaznu.kz

Нурланбек Ульпан – докторант 1-го года обучения по физике и астрономии, Казахский национальный университет им. Аль-Фараби, Алматы, Казахстан, e-mail: ulpan.nurlanbek02@gmail.com

Рабиғұлова Гүлдана – докторант 1-го года обучения по физике и астрономии, Казахский национальный университет им. Аль-Фараби, Алматы, Казахстан, e-mail: guldanaberikhanovna@gmail.com

Адил Махаббат – магистрант 1-го года обучения по физике и астрономии, Казахский национальный университет им. Аль-Фараби, Алматы, Казахстан, e-mail: adilmahabbat@gmail.com

Бекмурат Бағжан – студент 1-го курса магистратуры по физике и астрономии, Казахский национальный университет им. Аль-Фараби, Алматы, Казахстан, e-mail: bagzhan159@gmail.com

Тузен Галия – студент 1-го курса магистратуры по физике и астрономии, Казахский национальный университет им. Аль-Фараби, Алматы, Казахстан, e-mail: galiyatuzenn@gmail.com

Уразалина Айнур – PhD, доцент, Национальная открытая лаборатория нанотехнологий, Казахский национальный университет им. Аль-Фараби, Алматы, Казахстан, e-mail: y.a.a.707@mail.ru