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Immediate challenges faced by the quantum computing in time series analysis

We considered a few aspects of quantum computing in connection with the time series analysis. Quantum Fourier Transform was selected as a test example due to its important practical value in spectral analysis, the easiness of implementation and its generic nature with respect to many other quantum algorithms. The obvious drawbacks have been identified preventing the straightforward application of Quantum Fourier Transform to the evolving times series. The limited available register size of a quantum computer may be an issue at the data postprocessing stage, but carry significant practical value if included into the data acquisition stage. The analyzed qubit by qubit procedure is favoring the way most of the time series are acquired, which is one at a time. This procedure should be necessarily considered with the decoherence issue for the big quantum systems and long evolution times.

Keywords: quantum computing, qubit, time series, algorithm, Fourier transform.

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Вопросы применения квантовых вычислений в анализе временных рядов

Нами рассмотрены некоторые аспекты квантовых вычислений применительно к анализу временных рядов. Квантовое преобразование Фурье было выбрано в качестве инструмента анализа из-за его большого практического значения в спектральном анализе и его важной роли в формулировке других квантовых алгоритмов. Были обнаружены очевидные трудности в применении квантового преобразования Фурье к анализу временных рядов. Малый размер квантового регистра памяти накладывает серьезные ограничения при обработке большого массива уже зарегистрированных данных, но может давать существенное преимущество при непосредственном включении его в схему регистрации непрерывного потока данных. Используемая побитовая схема преобразования Фурье хорошо согласуется с последовательным механизмом регистрации временных рядов. Многочастичные квантовые системы и продолжительные времена эволюции, возникающие при реализации квантовых алгоритмов, накладывают ограничения на практическую реализацию этой схемы.

Ключевые слова: квантовые вычисления, кубит, временной ряд, алгоритм, Фурье преобразование.

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Уақыттық қатарларды талдауда кванттық есептеулерді қолдану мәселелері

Бұл мақалада уақыттық қатарларды талдаудағы кванттық есептеулердің бірқатар аспектілері қарастырылған. Фурьенің кванттық түрлендіруінің құрал ретінде таңдап алынған себебі - оның спектралды анализде үлкен тәжірибелік мәнге ие екендігі, әрі басқа да кванттық алгоритмдерді тұжырымдауда маңызды рөл атқаратындығы болып табылады. Уақыттық қатарларды талдауда Фурьенің кванттық түрлендіруін қолдану кезінде айқын кедергілер анықталды. Кванттық жады регистрінің шағын көлемді болуы тіркелген мәліметтердің үлкен массивін өңдегенде үлкен шектеулер қояды, алайда ол мәліметтерді тіркеу сұлбасына тікелей қосылған кезде, айтарлықтай

артықшылықтар бере алады. Қолданылып отырған Фурье түрлендіруінің әр бөлшек жұбына арналған сұлбасы уақытша қатарларды тіркеудің жүйелі механизмімен жақсы үйлеседі. Көпбөлшекті кванттық жүйелер мен эволюцияның жалғасу уақыттары осы схеманың тәжірибелік іске асуына шектеу қояды.
Түйін сөздер: кванттық есептеулер, кубит, уақыттық қатар, алгоритм, Фурье түрлендіруі.

Introduction

In the early 80s, Feynman [1, 2] outlined the computational capabilities of a quantum system. He showed that a group of computational problems exists and it could be addressed, in terms of success in finding solution, only by means of a quantum computer. The properties of such machine, the existence of a quantum mechanical simulator (or emulator) of the Turing machine [3], and its practical effectiveness was shown by Deutsch [4]. The following rapid growth of publications ad-dressing the algorithms and physical implementations of such computers followed, and is illustrated by the ScienceWatch.com web database maintained by Thomson Reuters [5]. Among these algo-rithms is the Quantum Fourier Transform (QFT). Though not attributed in name, partly due to its apparent simplicity, to anyone but a Fourier, it is an essential part of many other algorithms, includ-ing Shor's discrete logarithm and factoring algorithms [6, 7]. Besides well known classical applica-tions in a spectral analysis aside, Quantum Fourier Transform has its unique applications in phase estimation, order-finding and hidden subgroup problem [8].

The quantum Fourier transform (QFT) acts on a quantum state. Defined to reproduce the clas-sical discrete Fourier transform (FT) it says that any superposition of the basis states such as

$$\left|\psi\right\rangle = \sum_{j=0}^{N-1} x_j \left|j\right\rangle = \begin{pmatrix} x_0 \\ \vdots \\ x_{N-1} \end{pmatrix} \tag{1}$$

may experience the unitary transformation described by a matrix F, such as

$$F\left|\psi\right\rangle = \sum_{k=0}^{N-1} y_k \left|k\right\rangle, \text{ where } y_k = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} x_j e^{\frac{2\pi ijk}{N}} . \tag{2}$$

If we rewrite the phase factors $\omega=e^{\pi i/2}=i$, $\omega^2=e^{\pi i}=-1$ and etc, the 2-qubit quantum Fourier transform (for example the one acting on the bases |00⟩, |01⟩, |10⟩, |11⟩ of a two spin system) will be described by multiplication with the unitary matrix F of the following form

$$F = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & \omega & \omega^2 & \omega^3 \\ 1 & \omega^2 & 1 & \omega^2 \\ 1 & \omega^3 & \omega^2 & \omega \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{pmatrix} \tag{3}$$

which is a Vandermonde matrix. It is easier to work with the number of states N equals to some power *n* of *2*. If numbered in an ascending order these kets will be

$$\left|0\right\rangle, ..., \left|2^n - 1\right\rangle, \tag{4}$$

and they are constitute the computational basis for an n qubit quantum computer. If we to adopt the binary representation for integer number

$$j = j_1 j_2 ... j_3 \quad \text{or} \quad j = j_1 2^{n-1} + j_2 2^{n-2} + ... + j_n 2^0 , \tag{5}$$

and for binary fraction

$$0.j_l j_{l+1} ... j_m \quad \text{that is} \quad j_1 / 2 + j_{l+1} / 4 + ... + j_m / 2^{m-l+1} , \tag{6}$$

any QFT may be written as the operator product, see [9]

$$|\, j_1 j_2 ,..., j_3 \rangle \to \frac{(|0\rangle + e^{2\pi i 0.j_n} |1\rangle)(|0\rangle + e^{2\pi i 0.j_{n-1} j_n} |1\rangle) \cdots (|0\rangle + e^{2\pi i 0.j_1 j_2 \cdots j_n} |1\rangle)}{2^{\frac{n}{2}}} . \tag{7}$$

The described qubit by qubit procedure is favoring the way most of the time series are acquired, which is one at a time. This procedure should be necessarily considered with the decoher-ence issue for the big quantum systems and long evolution times. That is probably the only differ-ence that sets aside time series from the other data types and connects it to the QFT described by Eq.(7). It is also sufficient for real time data processing, where the whole array is not available for immediate analysis.

Experimental results and Discussions

Below is a data sample acquired by a single channel of Tian-Shian high elevation mounting station [10] in a period between November 21st, 2012 and November 30th, 2012.

The raw data of the period of about five days are shown in figures 1 (a). The Fourier trans-form coefficients of these data are shown to the right side of the figure, see Fig.1 b) and d). Anoth-er set, derived from original by the moving average and baseline subtraction, is shown in figure 1(c).

The first set of the Fourier coefficients looks more like a white noise than a signal. However, the FT of the cleaned data shows the expected one day periodicity, as well as the other features typ-ical for a discrete Fourier transform of the truncated data set. The 3, 5 and 7 days maxima are most probably influenced by data multiplication with the *rect* function of the period of 5 days together transformed by FT into convolution with a *sinc* function.

Let us assume now that we acquiring our data in small portions equal to the register size. Slid-ing window of acquisition is straightforwardly described by Eq.(8), where the width of the rect function is determined by the register size of a quantum computer

$$rect(t - \Delta t) \xrightarrow{FT} e^{-2\pi i \Delta t \xi} sinc(\xi) . \tag{8}$$

Everything else outside the sliding *rect(t-Δt)* function is zero, padded to the size of the final data set. Additional phase shift may be treated by a quantum circuitry as uninterrupted evolution in time though the phase control and readout remain an issue. The register size still needs to be big enough to accommodate the whole 5 days data chunk, which is about 15 thousands counts, one per each second.

An alternative method is called the overlap-add method [11]. The method still depends on the number of registers available, but it relies on, stitching together the pieces of FT on a smaller amount of data. In ideal reconstruction procedure we will end up no further than the scenario shown on Fig.1 c).

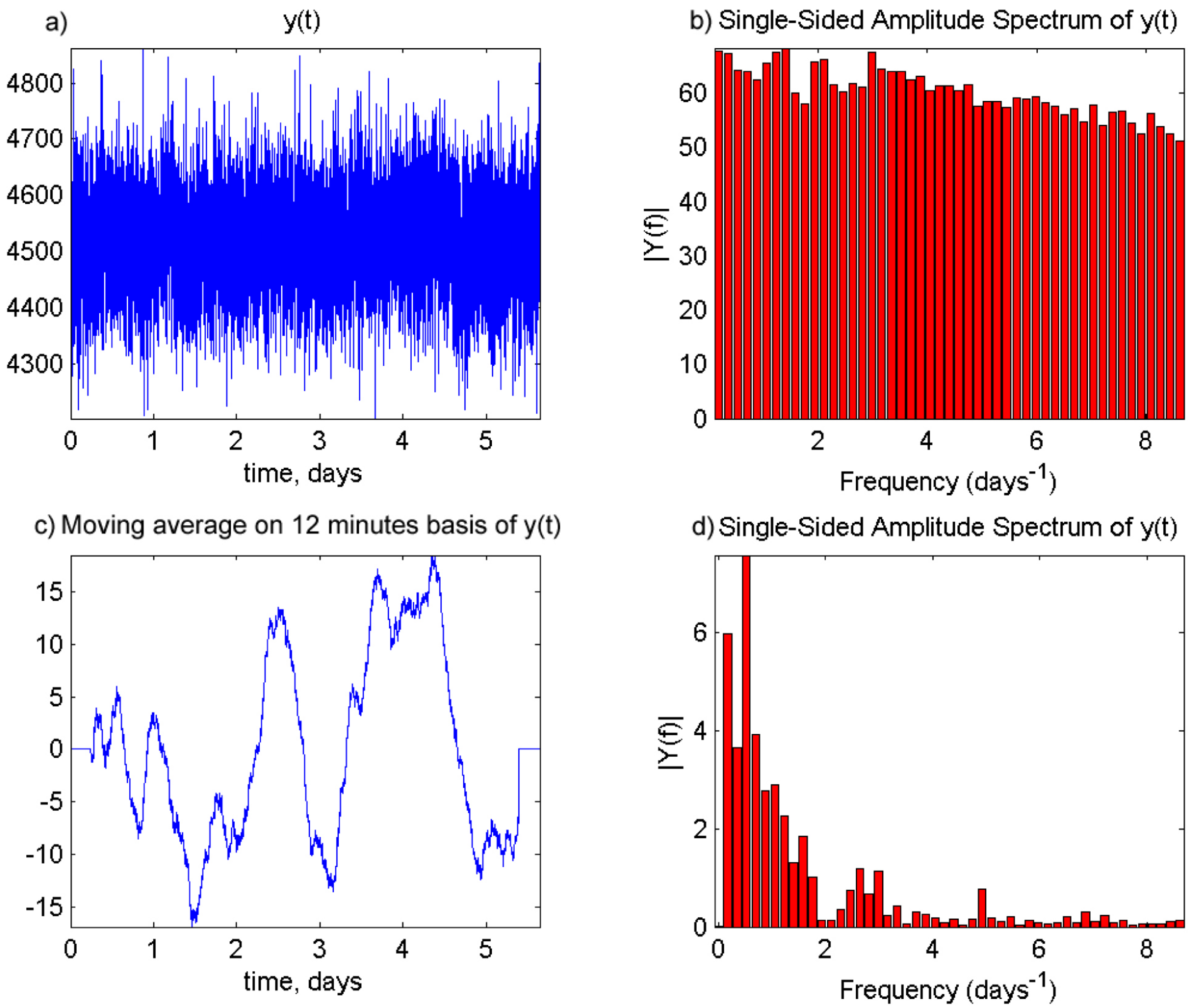

Figure 1 – From top to the bottom and from left to the right. a) Unfiltered five and a half days data from a single acquisition channel of 18NM64 neutron monitor (Y axis displays the neutron's counts), b) its Fourier transform coefficients' absolute values, c) original signal averaged and sub-tracted with the baseline (Y axis displays the arbitrary units) and d) its Fourier transform coeffi-cients' absolute values.

Conclusions

So far, the closest connection point with the quantum computing is its time delayed data availability matching the consecutive execution of the scheme described by Eq.(7). The straightforward approach by means of segmented or windowed Fourier transform most likely will not work if used in the final post processing stage. Acquisition process, that is detection and measurements, may be integrated into quantum state preparation stage. Further investigations in terms of incorporating it in the acquisition board is needed, and promises to be more fruitful. The search for applications based on a small amount of qubits available for computational tasks in time series analysis should be continued.

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