IRSTI 29.01.07; 29.17.01; 29.05.03

Abishev M.*, Toktarbay S., Khassanov M., Abylayeva A.

Al-Farabi Kazakh National University,
Kazakhstan, Almaty,*e-mail: abishevme@mail.ru

PROPAGATION OF A ELECTROMAGNETIC RADIATION IN THE STRONG MAGNETIC QUADRUPOLE AND GRAVITATIONAL FIELD

In the work, the nonlinear effect of the magnetic quadrupole field on the propagation of electromagnetic waves in the eikonal approximation of the parametrized post-Maxwell electrodynamics of the vacuum is calculated. Equations of motion **for** electromagnetic pulses transmitted in a strong magnetic field by two normal modes with mutually orthogonal polarization are constructed. The difference in propagation times of normal waves from the common source of electromagnetic radiation to the receiver is calculated. It is shown that the front and back parts of any hard radiation pulse due to the nonlinear electromagnetic influence of the magnetic quadrupole field turn out to be linearly polarized in mutually perpendicular planes, and the remaining part of the pulse must have elliptical polarization.

Key words: magnetic field, nonlinear electrodynamics, general relativity, polarization, quadrupole, electromagnetic radiation.

Әбішев М.*, Тоқтарбай С., Хасанов М., Абылаева А.

Әл-Фараби атындағы Қазақ ұлттық университеті,
Қазақстан, Алматы қ., *e-mail: abishevme@mail.ru

Күшті магниттік квадруполь және гравиталиялық өрістердегі электромагниттік сәуленің таралуы

Жұмыста вакуумдағы бейсызық параметрленген постмаксвеллдық электродинамикасының эйконалды жуықтауындағы электромагниттік толқындардың таралуына квадрупольді магнит өрісінің әсері есептелген. Күшті магнит өрісінде өзара ортогоналды поляризациясы бар екі нормалды модалармен берілген электромагниттік импульстердің қозғалыс теңдеулері тұрғызылды. Электромагниттік сәулеленудің жалпы көзінен қабылдағышқа дейінгі қалыпты толқындардың таралу уақытының айырмашылығы есептелді. Кез келген қатаң сәуле импульсінің алдыңғы және артқы бөліктері магниттік квадрупольдің сызықты емес электромагниттік әсерінен өзара перпендикулярлы жазықтықтарда сызықты түрде поляризацияланған, ал импульстің қалған бөлігі эллипстік поляризацияға ие болатыны көрсетілген.

Түйін сөздер: магнит өрісі, сызықты емес электродинамика, жалпы салыстырмалық теориясы, поляризация, квадруполь, электромагниттік сәулелену.

Абишев М.*, Токтарбай С., Хасанов М., Абылаева А.

Казахский национальный университет имени аль-Фараби,
Казахстан, г. Алматы, *e-mail: abishevme@gmail.com

Распространение электромагнитного излучения в сильном магнитном квадрупольном и гравитационном полях

В работе рассчитан нелинейный эффект магнитного квадрупольного поля на распространение электромагнитных волн в эйкональном приближении параметризованной постмаксвелловской электродинамики вакуума. Построены уравнения движения электромагнитных импульсов, передаваемых в сильном магнитном поле двумя нормальными модами с взаимно ортогональной поляризацией. Рассчитана разность времени распространения нормальных волн от общего источника электромагнитного излучения до приемника. Показано, что передние и задние части любого жесткого импульса излучения из-за нелинейного электромагнитного воздействия магнитного квадрупольного поля оказываются линейно поляризованными во взаимно перпендикулярных плоскостях, а оставшаяся часть импульса должна иметь эллиптическую поляризацию.

Ключевые слова: магнитное поле, нелинейная электродинамика, общая теория относительности, поляризация, квадруполь, электромагнитное излучение.

1. Introduction

According to the ideas of modern theoretical astrophysics [1-2], neutron stars have magnetic dipole fields, which on their surface reach values comparable with quantum electrodynamic induction $B_q = 4.41 \cdot 10^{13}$ Gs. In such fields, the nonlinearity of electrodynamics in a vacuum must appear, leading to the appearance of various physical effects [3-8]. Theoretical studies of such nonlinear electrodynamic processes use the post-Maxwellian approximation [9]. In this approximation, the Lagrangian of the nonlinear electrodynamics of vacuum is written in the parametrized form:

$$L == \frac{1}{32\pi}\left\{2J_2 + \xi\left[\left(\eta_1 - 2\eta_2\right)J_2^2 + 4\eta_2 J_4\right]\right\} - \frac{1}{c} j^m A_m ,$$

where $J_2 = F_{nk}F^{kn}$ and $J_4 = F_{nk}F^{km}F_{mi}F^{in}$ are invariants of the electromagnetic field tensor F_{kn}, $\xi = 1/B_q^2$, $\eta_{1,2}$ are postmaxwellian parameters whose magnitude is different in different theoretical models of nonlinear electrodynamics of vacuum.

2. The equations of the electromagnetic field

In the Heisenberg-Euler theory, which is a consequence of quantum electrodynamics, the numerical values **for** the parameters $\eta_1$and η_2 differ $\eta_1 = e^2/(45\pi\hbar c) = 5.1 \cdot 10^{-5}$, $\eta_2 = 7\, e^2/(180\pi\hbar c) = 9.0 \cdot 10^{-5}$, while in the nonlinear Born-Infeld electrodynamics they are equal to each other.

The equations of the electromagnetic **field with** have the form:

$$\frac{\partial}{\partial x^n}\{[1 + \xi(\eta_1 - 2\eta_2)J_2]F^{\mathrm{mn}} + 4\xi\eta_2 F^{\mathrm{mk}}F_{\mathrm{kp}}F^{\mathrm{pn}}\}$$
$$\mathrm{h} = -\frac{4\pi}{c} j^m. \qquad (1)$$

The second pair of equations of the electromagnetic field coincides with the corresponding equations of Maxwell's theory:

$$\frac{\partial F_{\mathrm{kn}}}{\partial x^m} + \frac{\partial F_{\mathrm{nm}}}{\partial x^k} + \frac{\partial F_{\mathrm{mk}}}{\partial x^n} = 0.$$

When studying the laws of propagation for weak electromagnetic waves in a strong external field F_{ik} the eikonal equation was used. Calculations have shown that the propagation of a weak electromagnetic wave according to the laws of nonlinear electrodynamics (1) in space-time with a metric tensor $g_{\mathrm{nk}}$and in the presence of an external electromagnetic field occurs by geodesic of some effective pseudo-Riemannian space-time. The metric tensor of this space-time G_{nk} depends on the metric tensor g_{nk}, the quadratic combination of the electromagnetic field tensor $F_{\mathrm{ni}}g^{\mathrm{im}}F_{\mathrm{mk}}$and at $\eta_1 \neq \eta_2$ it is different for waves of different polarization (nonlinear electrodynamic birefringence). While for the first normal wave the tensor G_{nk} has the form

$$G_{\mathrm{nk}}^{(1)} = g_{\mathrm{nk}} - 4\eta_1\xi F_{\mathrm{ni}}g^{\mathrm{im}}F_{\mathrm{mk}}, \qquad (2)$$

for the second normal wave, having orthogonal polarization to the polarization of the first wave, the tensor differs by the second term coefficient:

$$G_{\mathrm{nk}}^{(2)} = g_{\mathrm{nk}} - 4\eta_2\xi F_{\mathrm{ni}}g^{\mathrm{im}}F_{\mathrm{mk}}. \qquad (3)$$

According to the Lagrange-Charpy theorem, this means that in order to find the trajectories along which the momentum of a weak electromagnetic wave propagates in the external field and determine the laws of its motion along these trajectories, we need to solve the equations of isotropic geodesic motion in the effective space-time with the metric tensor $G_{\mathrm{nk}}^{(1,2)}$:

$$\frac{\mathrm{dK}^i}{d\Sigma} + \Gamma_{\mathrm{mn}}^i K^m K^p = 0, G_{\mathrm{nm}}^{(1,2)} K^n K^m = 0, \qquad (4)$$

where Γ_{mn}^i are Christoffel symbols of the space-time with a metric tensor $G_{\mathrm{nk}}^{(1)}$or $G_{\mathrm{nk}}^{(2)}$, depending on the mode being studied, $\Sigma$is affine parameter, $K^i = \mathrm{dx}^i/d\Sigma$ is a four-vector tangent to the corresponding isotropic geodesic.

3. Quadrupolar magnetic field components

We place the beginning of the Cartesian coordinate system at the point where the magnetic quadrupole is located. Then, the components of the magnetic induction vector **B** of this quadrupole in a spherical coordinate system r, θ, ϕ will have the form:

$$B_r = -\frac{\mathrm{BR}^4}{r^4}\left\{\left[\frac{1}{2}\sqrt{\frac{5}{2}}(1+3\cos 2\theta)\right]\cos\chi_1 - \left[3\sqrt{\frac{5}{6}}\sin 2\theta\cos\phi\right]\sin\chi_1\cos\chi_2 + \left[3\sqrt{\frac{5}{6}}\sin^2\theta\cos 2\phi\right]\sin\chi_1\sin\chi_2\right\}, - \left[3\sqrt{\frac{5}{6}}\sin 2\theta\cos\phi\right]\sin\chi_1\cos\chi_2 + \left[3\sqrt{\frac{5}{6}}\sin^2\theta\cos 2\phi\right]\sin\chi_1\sin\chi_2\Bigg\},$$

$$B_\theta = -\frac{\mathrm{BR}^4}{r^4}\left\{\left[\sqrt{\frac{5}{2}}\sin 2\theta\right]\cos\chi_1 + \left[10\sqrt{\frac{1}{30}}\cos 2\theta\cos\phi\right]\sin\chi_1\cos\chi_2 - -\left[\sqrt{\frac{5}{6}}\sin 2\theta\cos 2\phi\right]\sin\chi_1\sin\chi_2\right\},$$

$$B_\phi = \frac{\mathrm{BR}^4}{r^4}\left\{5\sqrt{\frac{2}{15}}\cos\theta\sin\phi\sin\chi_1\cos\chi_2 - \sqrt{\frac{10}{3}}\sin\theta\sin 2\phi\sin\chi_1\sin\chi_2\right\},$$

where R is the neutron star radius, B is the magnetic field at the stellar surface, $\chi_1[0,\pi]$ and $\chi_2[0,2\pi]$ are two angles specifying the particular geometry of the quadrupole magnetic field.

However, for further calculations it is more convenient for us to use a rectangular Cartesian coordinate system. Re-designating the constants B, χ_1 and χ_2 in accordance with relations

$$B_0 = B\sqrt{1+2\cos^2\chi_1},$$

$$\cos\xi = \frac{\sqrt{3}\cos\chi_1}{\sqrt{1+2\cos^2\chi_1}},$$

$$\sin\xi = \frac{\sin\chi_1}{\sqrt{1+2\cos^2\chi_1}},$$

we obtain:

$$B_x = \sqrt{\frac{5}{6}}\frac{B_0 R^4}{r^7}\{x[r^2-5z^2]f_1 + 2z[5x^2-r^2]f_2 + x[5z^2-3r^2+10y^2]f_3\},$$

$$B_y = \sqrt{\frac{5}{6}}\frac{B_0 R^4 y}{r^7}\{[r^2-5z^2]f_1 + 10\mathrm{xzf}_2 + [3r^2-10x^2-5z^2]f_3\},$$

$$B_z = \sqrt{\frac{5}{6}}\frac{B_0 R^4}{r^7}\{z[3r^2-5z^2]f_1 + 2x[5z^2-r^2]f_2 - 5z[x^2-y^2]f_3\},$$

where to abbreviate the notation: $f_1 = \cos\xi, f_2 = \sin\xi\cos\chi_2, f_3 = \sin\xi\sin\chi_2$, at that $f_1^2+f_2^2+f_3^2 = 1$.

Suppose that an electromagnetic pulse is emitted from a certain point $\mathbf{r} = \mathbf{r}_s = \{x_s, y_s, z_s\}$ at time $t = t_s$. We assume that at the point $\mathbf{r} = \mathbf{r}_d = \{x_d, y_d, z_d\}$ there is an electromagnetic radiation detector. We orient the axes of the Cartesian coordinate system so that the source and the electromagnetic radiation detector lie in the XOZ plane, and the Z axis is directed so that the following conditions are fulfilled: $x_s = x_d$, $y_s = y_d = 0$. Then $\mathbf{r}_s = \{x_s, 0, z_s\}$, *and* $\mathbf{r}_d = \{x_s, 0, z_d\}$. As in [19], we will consider the propagation of pulses of only X-ray and gamma frequencies, for which the influence of the magnetosphere of a pulsar and a magnetar can be neglected.

Let us find out by which rays the electromagnetic pulses will propagate from the point $\mathbf{r}_s$ to the point $\mathbf{r}_d$, and also determine the laws of motion of electromagnetic pulses along these rays.

We find the components of metric tensors $G_{\rm nk}^{(1,2)}$ of the effective pseudo-Riemannian space-time (3) - (4) for the problem under consideration:

$$G_{00}^{(1,2)} = 1\,, G_{\alpha\beta}^{(1,2)} =$$
$$= -\delta_{\alpha\beta}\left[1 + 4\eta_{1,2}\,\xi B^2(r)\right] + 4\eta_{1,2}\xi B_\alpha(r) B_\beta(r).$$

The vector **B** of the magnetic quadrupole entering into these expressions must be taken with the Maxwellian accuracy.

The equations of geodesics in space-time (2) - (3), can be written by differentiating not with respect to the parameter Σ, but in the coordinate z in accordance with expression $d/d\Sigma = K^3\, d/\mathrm{dz}$.

4. Calculation of the delay time

Our equations are nonlinear, for which the usual methods of integration are not applicable. However, they contain a small parameter $\xi\, B_0^2$. Therefore, we represent expressions $x = x_{1,2}(z), y = y_{1,2}(z)$and $t = t_{1,2}(z)$ in the form of expansions with respect to this small parameter:

$$t_{1,2}(z) = t_0(z) + \eta_{1,2}\xi B_0^2[t(z) - t(z_s)],$$

$$x_{1,2}(z) = x_0(z) + \eta_{1,2}\xi B_0^2\left[X(z) - X(z_s) + \frac{(z-z_s)\,[X(z_s) - X(z_d)]}{(z_d - z_s)}\right],$$

$$y_{1,2}(z) = y_0(z) + \eta_{1,2}\xi B_0^2\left[Y(z) - Y(z_s) + \frac{(z-z_s)\,[Y(z_s) - Y(z_d)]}{(z_d - z_s)}\right].$$

Since the electromagnetic pulse at time $t = t_s$ was at the point $\mathbf{r} = \mathbf{r}_s$, and the ray must pass through the points $\mathbf{r} = \mathbf{r}_s$ и $\mathbf{r} = \mathbf{r}_d$, we obtain:

$$x_0(z_s) = x_0(z_d) = x_s, y_0(z_s) =$$
$$= y_0(z_d) = 0, t_0(z_s) = t_s.$$

Then in the Maxwellian approximation we will have:

$$c\frac{d^2 t_0(z)}{\mathrm{dz}^2} = \frac{d^2 x_0(z)}{\mathrm{dz}^2} = \frac{d^2 y_0(z)}{\mathrm{dz}^2} = 0,$$

$$c^2\left(\frac{\mathrm{dt}_0(z)}{\mathrm{dz}}\right)^2 - \left(\frac{\mathrm{dx}_0(z)}{\mathrm{dz}}\right)^2 - \left(\frac{\mathrm{dy}_0(z)}{\mathrm{dz}}\right)^2 = 1.$$

From these equations it follows that:

$$t_0(z) = t_s + \frac{z - z_s}{c}, x_0(z) = x_s, y_0(z) = 0$$

We have:

$$\frac{\mathrm{dt}(z)}{\mathrm{dz}} = \frac{2}{c}\{B_x^2 + B_y^2\} = \frac{5R^8}{3c}\left\{\frac{25x_s^5}{r^{14}}\,[4\mathrm{zf}_2(f_1 - f_3) + x_s(1 - 2f_1 f_3 - 5f_2^2)] + \right.$$
$$+\frac{20x_s^3}{r^{12}}\,[\mathrm{zf}_2(3f_3 - 5f_1) + x_s(f_3^2 + 9f_2^2 + 3f_1 f_3 - 2)] + \frac{4f_2^2}{r^8} +$$
$$\left. + \frac{4x_s}{r^{10}}[2\mathrm{zf}_2(2f_1 - f_3) + x_s(4 - 4f_1 f_3 - 15f_2^2 - 3f_3^2)]\right\},$$

where in the approximation under consideration $r = \sqrt{z^2 + x_s^2}$.

Integrating this equation, we find the dependence of $t(z)$:

$$t(z)=\frac{5R^8}{6c}\left\{\frac{50x_s^5}{3r^{12}}f_2(f_3-f_1)-\frac{8f_2^2z}{7r^8}+\frac{4x_s^3}{r^{10}}f_2(5f_1-3f_3)+\frac{x_s}{r^8}f_2(2f_3-4f_1)+\right. \tag{13}$$

$$+\frac{25x_s^4z}{6r^{12}}(1-5f_2^2-2f_1f_3)-\frac{x_s^2z}{12r^{10}}(41-48f_3^2-157f_2^2-34f_1f_3)+$$

$$\left.+\frac{5}{512x_s^7}\left[atan\left(\frac{z}{x_s}\right)+\frac{zx_s}{r^2}+\frac{2zx_s^3}{3r^4}+\frac{8zx_s^5}{15r^6}+\frac{16zx_s^7}{35r^8}\right][35-182f_1f_3+193f_2^2+336f_3^2]\right\}$$

For the functions *X (z)* and *Y (z)* we obtain the following equations:

$$\frac{d^2X(z)}{dz^2}=\frac{1750R^8x_s^6}{3r^{16}}\{4zf_2(f_1-f_3)+x_s(f_1^2-2f_1f_3-4f_2^2+f_3^2)\}+$$

$$+\frac{100R^8x_s^4}{3r^{14}}\{z(83f_3-107f_1)f_2+x_s(62f_1f_3-37f_1^2+136f_2^2-25f_3^2)\}+$$

$$+\frac{100R^8x_s^2}{3r^{12}}\{z(43f_1-13f_3)f_2+x_s(25f_1^2-28f_1f_3-76f_2^2+5f_3^2)\}+$$

$$+\frac{40R^8}{3r^{10}}\{x_s(f_3^2+5f_1f_3-14f_1^2+21f_2^2)-z(6f_1+f_3)f_2\},$$

$$\frac{d^2Y(z)}{dz^2}=0$$

Integrating these equations, we find:

$$X(z)=\frac{25R^8}{3072x_s^8}\{z(1274f_1f_3-1596f_2^2-245f_1^2-2597f_3^2)-$$

$$+\frac{5R^8x_s^2}{72r^{10}}\{16z(13f_1-7f_3)f_2+x_s(83f_1^2-118f_1f_3-284f_2^2+35f_3^2)\}+$$

$$+-64x_s(f_1+13f_3)f_2\}atan\left(\frac{z}{x_s}\right)+\frac{125R^8x_s^4}{36r^{12}}\{4z(f_3-f_1)f_2-$$

$$+\frac{5R^8}{576r^8}\{x_s(145f_3^2+524f_2^2+46f_1f_3-287f_1^2)-192z(f_1+2f_3)f_2\}+$$

$$+\frac{5R^8}{1152x_s^2r^6}\{x_s(35f_1^2+228f_2^2-182f_1f_3+371f_3^2)-64z(f_1+13f_3)f_2\}+$$

$$+\frac{25R^8}{4608x_s^4r^4}\{x_s(245f_1^2-1274f_1f_3+1596f_2^2+2597f_3^2)-320z(f_1+13f_3)f_2\}+$$

$$+\frac{25R^8}{9216x_s^6r^2}\{x_s(245f_1^2-1274f_1f_3+1596f_2^2+2597f_3^2)-192z(f_1+13f_3)f_2\},$$

$$Y(z)=0.$$

Let us consider the effects of the nonlinear electrodynamic action of the magnetic quadrupole field on the electromagnetic wave.

Estimates show that when $B_0 \sim 10^{13}$ G the angle β can reach several angular seconds. However, because of the large distance between pulsars and the Earth, compared with the radii of pulsars, the angles of non-linear electrodynamic curvature of rays from the solar system can not be measured.

Further, for $\eta_1 \neq \eta_2$ because of the nonlinear-electrodynamic birefringence, each electromagnetic pulse emitted at the point $\mathbf{r}_0 = \{q, 0, z_0\}$, splits into two pulses, one of which is carried by the first normal wave and the other by the second normal wave having orthogonal polarization. These pulses move to the receiver along different beams, spending on this way different time.

We calculate the delay time of the electromagnetic pulse carried by the first normal wave, in comparison with the propagation of the momentum carried by the second normal wave.

$$\Delta t = \frac{25\pi(\eta_1 - \eta_2)\, \xi B_0^2 R^8}{3072 x_s^7 c} \times$$

$$\times \left[35 - 182 f_1 f_3 + 193 f_2^2 + 336 f_3^2\right]$$

The presence of a non-zero value of Δt leads to the appearance of unusual polarization properties for an electromagnetic pulse. These properties are a consequence of the different magnitude of the propagation velocity of two modes in an external magnetic field. Indeed, suppose that a pulse of an arbitrarily polarized hard radiation of finite duration T. Because of the birefringence of the vacuum, it splits into two modes, polarized in mutually perpendicular planes, with the leading edges of these modes coinciding at the initial instant of time. The leading edge of the faster mode will arrive at the hard radiation detector earlier than the leading edge of the slow mode, which for some time is equal to Δt. Therefore, during the time Δt, only the faster normal pulse mode will pass through the detector and the detector will detect the linear polarization of this part of the momentum.

After the time Δt, the front of the momentum transferred by another normal mode, the phase of which differs from the phase of the faster mode on $\omega \Delta t$, where ω is the frequency of the wave. The addition of these orthogonally polarized normal modes in the subsequent time will create in the detector radiation with elliptical polarization that will pass through the detector for a time $T - \Delta t$.

Quite analogously, the trailing edge of the faster momentum mode will leave the detector before the trailing edge of the slow mode. Therefore, at the back of the hard radiation momentum duration Δt, the polarization will also be linear, but orthogonal to the linear polarization of the front of the momentum.

Thus, the detection of the above-mentioned polarization properties of hard pulses coming from pulsars makes it possible not only to assert the manifestation of nonlinear electrodynamics of vacuum, but also to estimate the magnitude of the induction of the magnetic field on the surface of the pulsar from the value of Δt.

5. Conclusion

The calculation showed that, according to the equations of nonlinear electrodynamics of vacuum, the magnetic quadrupole field bends the rays of electromagnetic waves and the magnitude of the angle of this curvature depends on the orientation of the quadrupole moment with respect to the direction of propagation of the electromagnetic wave.

The propagation velocities of electromagnetic pulses at $\eta_1 \neq \eta_2$ depend on the polarization of the electromagnetic wave. If a short pulse is emitted from the electromagnetic radiation source, then in the general case it will propagate in the magnetic quadrupole field in the form of two normal waves having mutually perpendicular polarization.

In the receiver of electromagnetic radiation, these pulses will arrive along different beams and at different instants, as a result of which the recorded total pulse will have an unusual polarization: the front and back parts of each pulse of length $c\Delta t$ will be linearly polarized in mutually perpendicular planes, and the part momentum will be elliptically polarized. A simple analysis shows that at $x_s \sim R \sim$ 10 km and $B_0 \sim 10^{13}$ G the value Δt with a favorable orientation of the quadrupole relative to the z axis of the Cartesian coordinate system chosen by us, can reach several tens of nanoseconds.

References

1 Abishev M., Aimuratov Y., Aldabergenov Y., Beissen N., Zhami B., Takibayeva M. Some astrophysical effects of nonlinear vacuum electrodynamics in the magnetosphere of a pulsar // Astroparticle Physics. – 2016. – Vol.73. – P. 8-13.

2 Kaspi V.M., Lackey J.R., Mattox J., Manchester R.N., Bailes M. and Pace R. High-energy gamma-ray observations of two young, energetic radio pulsars // Astrophysical Journal. – 2000. – Vol. 528. – P.445.

3 Caniulef D.G., Zane S., Taverna R., Turolla R., Wu K. // MNRAS. – 2016. – Vol. 459. – P.3585.

4 Denisov V.I., Sokolov V.A. Analysis of regularizing properties of nonlinear electrodynamics in the Einstein-Born-Infeld theory // Journal of Experimental and Theoretical Physics. – 2011. – Vol. 113. – P. 926-933.

5 Denisov V.I., Sokolov V.A., Svertilov S.I. Vacuum non-linear electrodynamic polarization effects in hard emission of pulsars and magnetars // JCAP. – 2017. – Vol.09. – P.004

6 Epstein R., Shapiro I.I. Post-post-Newtonian deflection of light by the Sun // Phys. Rev. D. – 1980. – Vol. 22. – P.2947; Heisenberg W., Euler H. // Z. Phys. – 1936. – Vol.26. – P.714.

7 Kim J.Y. // JCAP. – 2011. – Vol.11. – P.056; Landau L.D., Lifshitz E.M. The Classical Theory of Fields. Pergamon Press, Oxford Manchester R.N., Taylor J.H., 1977, Pulsars. 1971.

8 Denisov V.I., Sokolov V.A., Vasili'ev M.I. Nonlinear vacuum electrodynamics birefringence effect in a pulsar's strong magnetic field // Phys. Rev. D. – 2014. – Vol. 90. – P.023011.

9 Denisov V.I., Denisova I.P., Pimenov A.B., Sokolov V. Rapidly rotating pulsar radiation in vacuum nonlinear electrodynamics // Eur. Phys. J. C. – 2016a. – Vol.76. – P. 612.

10 Freeman W.H. San Francisco Manchester R.N., Hobbs G. B., Teoh A., Hobbs M., 2005, AJ, 129, 1993

11 Mathews L.D., Walker R.L. Mathematical Methods of Physics. – New York, W.A. Benjamin, 1970.

12 Benjamin W.A., New York Mignani R. P., Testa V., Caniulef D. G., Taverna R., Turolla R., Zane S., Wu K. // MNRAS. – 2017. – Vol. 465. – P.492.

13 Olausen S.A., Kaspi V.M. The McGILL magnetar catalog // ApJS. – 2014. – Vol.212. – P.1-22; P´etri J. // MNRAS. – 2013. – Vol. 433. – P.986.

14 Denisov V.I., Shvilkin B.N., Sokolov V.A., Vasili'ev M.I. Pulsar radiation in post-Maxwellian vacuum nonlinear electrodynamics // Phys. Rev. D. – 2016. – Vol. 94. – P.045021.

15 Denisov V. I., Dolgaya E.E., Sokolov V.A. Nonperturbative QED vacuum birefringence // JHEP. – 2017. – Vol.105. – P.1.

16 P´etri J. Multipolar electromagnetic fields around neutron stars: exact vacuum solutions and related properties // MNRAS. – 2015. – Vol.450. – P.714-742.

17 Soffitta P. et al. XIPE: the X-ray imaging polarimetry explorer // Exp. Astron. – 2013. – Vol.36. – P.523-567.

18 Taverna R., Muleri F., Turolla R., Soffitta P., Fabiani S., Nobili L. Probing magnetar magnetosphere through X-ray polarization measurements // MNRAS. – 2014. – Vol.438. – P.1686-1697.

19 Vasili'ev M.I., Denisov V.I., Kozar' A.V., Tomasi-Vshivtseva P.A. The Effects of Vacuum Nonlinear Electrodynamics in a Electric Dipole Field // Moscow University Physics Bulletin. – 2017. – Vol.72. – P.513-517.

20 Weisskopf M. C. et al., 2016 , in Jan-Willem den Herder A., Tadayuki T., Marshall B., eds. // Proc. SPIE Conf. Ser. 9905, Ultraviolet to Gamma Ray. SPIE, Bellingham, 990517

21 Zhukovsky K.V. Solving evolutionary-type differential equations and physical problems using the operator method // Theoretical and Mathematical Physics. – 2017. – Vol.190. – P.52-68.

22 Zhukovsky K.V. Operational solution for some types of second order differential equations and for relevant physical problems // J. Math. Anal. Appl. –2017. – Vol.446. – P.628-647.

References

1 M. Abishev, Y. Aimuratov Y. Aldabergenov, N. Beissen, B. Zhami, M.Takibayeva, Astroparticle Physics, 73, 8-13 (2016).

2 V.M. Kaspi, J.R. Lackey, J. Mattox, R.N. Manchester, M. Bailes and R. Pace, Astrophysical Journal 528, 445 (2000)

3 D.G. Caniulef, S. Zane, R. Taverna, R. Turolla, K. Wu, MNRAS, 459, 3585 (2016).

4 V.I. Denisov, V.A. Sokolov, Journal of Experimental and Theoretical Physics, 113, 926 (2011).

5 V.I. Denisov, V.A. Sokolov, S.I. Svertilov, JCAP, 09, 004 (2017).

6 R. Epstein, I.I. Shapiro, Phys. Rev. D 22, 2947 1980; W. Heisenberg, H. Euler, Z. Phys., 26, 714 (1936).

7 J.Y .Kim, JCAP, 11, 056 (2011); L.D. Landau, E.M. Lifshitz, The Classical Theory of Fields. (Pergamon Press, Oxford Manchester R.N., Taylor J.H., 1977, Pulsars).

8 V.I. Denisov, V.A. Sokolov, M.I. Vasili'ev, Phys. Rev. D 90, 023011 (2014).

9 V.I. Denisov, I.P. Denisova, A.B. Pimenov, and V. Sokolov, Eur. Phys. J. C76, 612 (2016).

10 W.H. Freeman, San Francisco Manchester R.N., Hobbs G.B., Teoh A., Hobbs M., 2005, AJ, 129, 1993

11 L.D. Mathews, R.L. Walker Mathematical Methods of Physics, (New York, W.A. Benjamin, 1970).

12 W.A. Benjamin, New York Mignani R. P., Testa V., Caniulef D. G., Taverna R., Turolla R., Zane S., Wu K., MNRAS, 465, 492 (2017).

13 S.A. Olausen and V.M. Kaspi, ApJS, 212, 1-22 (2014); J. P´etri, MNRAS, 433, 986 (2013).

14 V.I. Denisov, B.N. Shvilkin, V.A. Sokolov, and M.I. Vasili'ev, Phys. Rev. D, 94, 045021 (2016).

15 V.I. Denisov, E.E. Dolgaya, and V.A. Sokolov, JHEP, 105, 1 (2017).

16 J. P´etri, MNRAS, 450, 714-742 (2015).
17 P. Soffitta et al., Exp. Astron., 36, 523-567, (2013).
18 R. Taverna, F. Muleri, R. Turolla, P. Soffitta, S. Fabiani, and L. Nobili, MNRAS, 438, 1686-1697 (2014).
19 M.I. Vasili'ev, V.I. Denisov, A.V. Kozar', and P.A. Tomasi-Vshivtseva, Moscow University Physics Bulletin, 72, 513-517 (2017).
20 M.C. Weisskopf et al., in Jan-Willem den Herder A., Tadayuki T., Marshall B., eds., Proc. SPIE Conf. Ser. 9905, Ultraviolet to Gamma Ray. SPIE, Bellingham, 990517 (2016).
21 K.V. Zhukovsky, Theoretical and Mathematical Physics, 190, 52-68 (2017).
22 K.V. Zhukovsky, Math. Anal. Appl., 446, 628-647 (2017).