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Energy spectrum of the quantum vortices configurations

A review of various exactly solvable models on the determination of the energy spectra E (k) of 3D-velocity field, induced by chaotic vortex lines is proposed. This problem is closely related to the sacramental question whether a chaotic set of vortex filaments can mimic the real hydrodynamic turbulence [1]. The quantity v(k)v(-k) can be exactly calculated, provided that we know the probability distribution functional P({s(\xi ,t)}) of vortex loops configurations

Key words: energy spectrum, the vortex lines, hydrodynamic turbulence, Kolmogorov spectrum.

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Энергетический спектр квантовых вихревых конфигураций

В работе предлагается обзор различных моделей, имеющих точное решение по определению энергетического спектра E(k) поля 3D-скорости, вызванного хаотичными вихревыми линиями. Эта проблема тесно связана с сакраментальным вопросом, может ли хаотическое множество вихревых линий имитировать реальную гидродинамическую турбулентность. Мы вводим общий метод расчета энергетического спектра по конфигурациям вихревых линий, рассматриваем несколько простых, но полезных примеров – прямая линия и вихревое кольцо радиуса R.

Ключевые слова: энергетический спектр, вихревые линии, гидродинамическая турбулентность, спектр Колмогорова.

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Кванттық құйын тәрізді конфигурациялардың энергетикалық спектрі

Бұл жұмыста құйын тәрізді хаосты сызықтардың әсерінен туындайтын 3D-жылдамдықты өрісінің энергетикалық спектрі E(k)бойынша нақты шешімі бар әртүрлі модельдерді қарастыру ұсынылады. Бұл мәселе көптеген құйын тәрізді хаосты сызықтар шынайы гидродинамикалық турбуленттілікке ұқсайды ма деген киелі сұрақпен байланысты. Біз құйын тәрізді сызықтардың конфигурациясы бойынша энергетикалық спектрінің есептеу әдістемесін енгіземіз, бірнеше қарапайым, бірақ пайдалы мысалдарды – түзу сызық және радиусы R құйын тәрізді шеңберді қарастырамыз.

Түйін сөздер: энергетикалық спектр, құйын тәрізді сызықтар, гидродинамикалық турбуленттілік, Колмогоров спектрі.

A review of various exactly solvable models on the determination of the energy spectra E (k) of 3D-velocity field, induced by chaotic vortex lines is proposed. This problem is closely related to the sacramental question whether a chaotic set of vortex filaments can mimic the real hydrodynamic turbulence [1]. The quantity $v(k)v(-k)$ can be exactly calculated, provided that we know the probability distribution functional $P(\{s(\xi ,t)\})$ of vortex loops configurations [2,3]. The knowledge of $P(\{s(\xi ,t)\})$ is identical to the full solution of the problem of quantum turbulence and, in general, $P(\{s(\xi ,t)\})$ unknown. In the paper we discuss several models allowing to evaluate spectra in the explicit form. These cases include the standard vortex configurations such as a straight line, vortex array and ring. Independent chaotic loops of various fractal dimension as well as interacting loops in the thermodynamic equilibrium also permit an analytical solution. We also describe the method of an obtaining the 3D velocity spectrum induced by the straight line perturbed with chaotic Kelvin waves on it. Especial attention will be paid to the spectrum produced by the collapsing and reconnected lines. It is shown that reconnecting lines generate spectrum *E* (*k*) close to the famous Kolmogorov spectrum $E(k) = k^{-5/3}$. These are works, based on both the vortex filament method and the Gross-Pitaevskii equation. The quantity v(k)v(-k) can be exactly calculated, provided that we know the probability distribution functional P({s(\xi ,t)}) of vortex loops configurations [3,4]. The knowledge of P({s(\xi ,t)}) is identical to the full solution of the problem of quantum turbulence and, in general, P({s(\xi ,t)}) unknown.

In the work we introduce the general method for calculation of the energy spectrum via the vortex line configuration, then we consider the couple of simple but useful examples -the straight line and vortex ring of radius R. Then we study uniform and nonuniform vortex arrays, the straight line with excited Kelvin wave on it and then we study the case of the reconnecting vortex filaments. We demonstrate that the spectra E(k), generated by the these configurations, is very close to the Kolmogorov dependence varproptok$^{-5/3}$, and discuss the reason for this as well as the reason for deviation.

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