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Electron density of states and localization of two-dimensional disordered systems in quantized magnetic fields

We study numerically non-interacting electrons moving on a two-dimensional lattice with a uniform magnetic field and a random site potential. The electron localization and the density of states are investigated by using the method of transfer-matrices and by the direct diagonalization technique. For numerical simulations the Ando model with the diagonal disorder is used. The first preliminary data have been obtained for different sizes of the system and various values of the magnetic field. The localization length exhibits Shubnikov-de-Haas oscillations. The density of states shows several Landau bands separated by the energy gaps. With increasing the disorder the Landau bands becomes broader and overlap with each other. The application of the obtained results to the integer quantum Hall effect is discussed.

Key words: electron conductivity, critical phenomena, two-dimensional electron gas, quantum Hall effect, electron localization

Introduction

The prediction of the absence of the delocalized states for non-interacting electrons in a disordered two-dimensional system in the limit of the vanishing magnetic field has been made in a seminal paper of “gang of four” [1]. This has been a birth of the celebrating scaling theory in condensed matter physics. Later on Klaus von Klitzing and coworkers have discovered a quantum Hall effect [2]. The key point of this phenomenon is a quantization of the Hall resistance occurring at very low temperatures close to the absolute zero and in an extremely high magnetic field. For this discovery von Klitzing has been honored by the Nobel price for physics in 1984.

This latter event has been followed by the discovery of the fractional quantum Hall effect (FQHE), observed experimentally by Tsui, Störmer and Gossard [3]. The FQHE has also won the Nobel Price in 1999 jointed by a theoretician Prof. Robert Laughlin who has elaborated an analytical explanation for the fractional version of the effect. That is why the discovery made by K. von Klitzing is referred to as the integer quantum Hall effect, i.e. IQHE (or ordinary QHE, respectively). IQHE was found in two-dimensional (2D) electron or hole layers of the spatial charge in the metal-insulator-semiconductor structures (MIS) and in the heterojunctions with modulated doping.

The quintessence of the effect is that in the low temperature Hall conductivity σ_H of the 2D degenerated electron gas in a strong magnetic field $\boldsymbol{B}$, which is measured as a function of magnetic field intensity B or of the concentration of the two-dimensional carriers N_s, one can observe a number of plateaus. This is shown in Fig. 1 as an example of the dependence of the Hall resistance R_H on the concentration N_s. The relation $R_H = 1/\sigma_H$ is valid in the region of a plateau. Exactly on the plateau we have

$$\sigma_H = \nu\, e^2/h. \qquad (1)$$

Here e and h are fundamental constants, i.e. the elementary charge and Planck’s constant, respectively, ν is filling factor which is equal to $\nu = N_s/ N_L$, with N_L being the number of the electron states in the Landau level measured per square unit.

$$N_L = 1/ 2\pi\lambda^2 = eB/ch = B/\Phi_0, \qquad (2)$$

where c is the light speed and $\lambda=(ch/eB)^{1/2}$ is the magnetic length and $\Phi_0 = ch/e$ is the magnetic flow quantum. Fo the first time the observation of the quantum Hall plateaus have been observed and measured in the Si-MOS right-angle structure at the temperature T=1.5 K [1] as a function of the

gate voltage V_G. The latter is proportional to the filling factor N_s.The similar "jumping" picture has been obtained later on the heterostuctures $GaAs/Al_{0.3}Ga_{0.7}As$ as a direct field dependence of the Hall resistance tensor ρ_{xy} at the temperature T=8 mK [4,5].

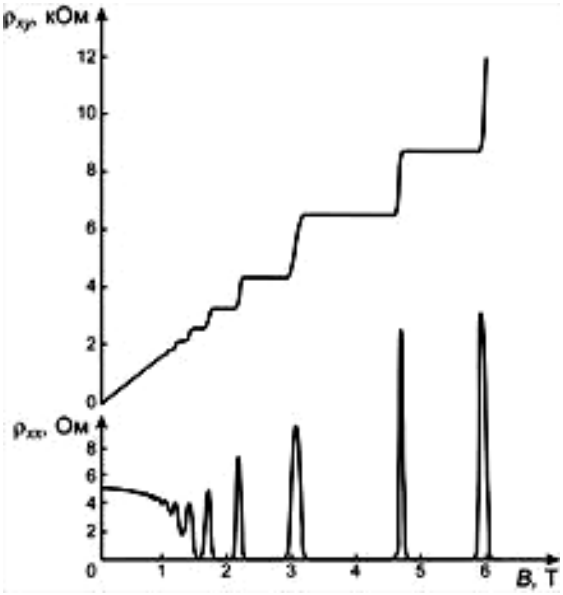

Fig.1. - The quantum Hall effect in the GaAs heterostructure. The dependence of the tensor of the Hall resistance: the transversal ρ_{xy} (on the top) and the longitudinal ρ_{xx} (on the bottom) on the strength of the external magnetic field B. The temperature is equal to T=8 mK [5]

Model of the numerical simulations

The study of electronic states in disordered systems is inevitable to understand electronic transport properties in conducting materials. Most directly the electronic states are described by their wave functions. Since the wave functions and the energies are both obtained by solving the eigenvalue problem for the disordered Hamiltonian, it is quite natural to expect that the energy spectrum involves some information on the eigenfunctions. In disordered systems the spatial symmetries which exist in pure systems are completely lifted. The fundamental symmetries under the operation of the time reversal can persist. The importance of them in the problem of the Anderson localization were first pointed out by Wegner [6,7] in the treatment of weakly localized regime. The link between the level statistics and the fundamental symmetry was originally proposed in nuclear physics in order to explain complicated energy spectra in some heavy nuclei [8-11], and then it has been applied to the problems of metallic fine particles [12, 13] and quantum chaos [14,15].

Although substantial progress in the understanding of critical behaviour at the disorder-induced IQHE-to-insulator transition has been achieved, nevertheless several issues are still considered as being open and unsolved. The first issue of them is knowledge of the density of states. The second one is the electron localization in the centre of the Landau bands. We use the famous Ando model with the diagonal disorder [16]

$$\mathrm{H} = \sum_r \in_r |r><r| + \sum_{\Delta} t_{r,r+\Delta} |r><r+\Delta|. \qquad (3)$$

The onsite energies are measured in units of the hopping integral $t_{r,r+\Delta}$, while the length scale is measured in the units of the lattice constant, i.e. a=1. The electron states denoted by $|\boldsymbol{r}>$ correspond to the lattice sites of the simple square lattice. The random energies ϵ_r are governed by the following distribution law

$$P(\varepsilon)=\begin{cases} W^{-1}, & |v|\le W/2, \\ 0, & |v|>W/2. \end{cases} \tag{4}$$

The hopping elements between neighboring states $\boldsymbol{r}$ and $\boldsymbol{r}+\Delta$ are considered and have the following form [17,18]:

$$t_{\mathrm{r,r}+\Delta}=\begin{cases} \exp(\mp 2\pi i\alpha z), & \Delta\in\{\pm e_y\}, \\ 1, & \Delta\in\{\pm e_x,\pm e_z\} \end{cases} \tag{5}$$

where the magnetic field B enters the problem through the factor $\alpha = N_L = \dfrac{eB}{hc}$, for simplicity we choose the standard gauge, namely $\vec{A}=(0,-B_z,0)$.

The density of states

We study the single-electron density of states of two-dimensional disordered systems in the presence of quantized magnetic field under various conditions. According to the definition of the density of states

$$\rho(E)=\frac{1}{L^2}\sum_n \delta(E-\varepsilon_n) \tag{6}$$

Here $\rho(E)$ is a global quantity, which is averaged of the whole surface of the two-dimensional electron gas. The discrete energies have been calculated by the straightforward diagonalisation of the Ando Hamiltonian (3). Figures 1 and 2 demonstrate the spectral density of states $\rho(E)$ at various disorder W of the random potential for the 2D system of linear size L=50 with fixed magnetic filed α=0.1 and α=0.01, respectively. The plots exhibit periodic behaviour, which is typical for the discrete lattice model. In both figures one observes the oscillating behaviour of the density of states. Close to the band edge the Landau bands are well separated by the energy gaps. With increasing the disorder W the Landau bands start to overlap with each other, their heights diminish. Finally, for larger W the gaps disappear entirely. Although the shapes of the density of states in Figure 1 and in the inset of Figure 2 are similar to each other, nevertheless $\rho(E)$ can not be mapped by a scaling transformation $f(\alpha,W)$.

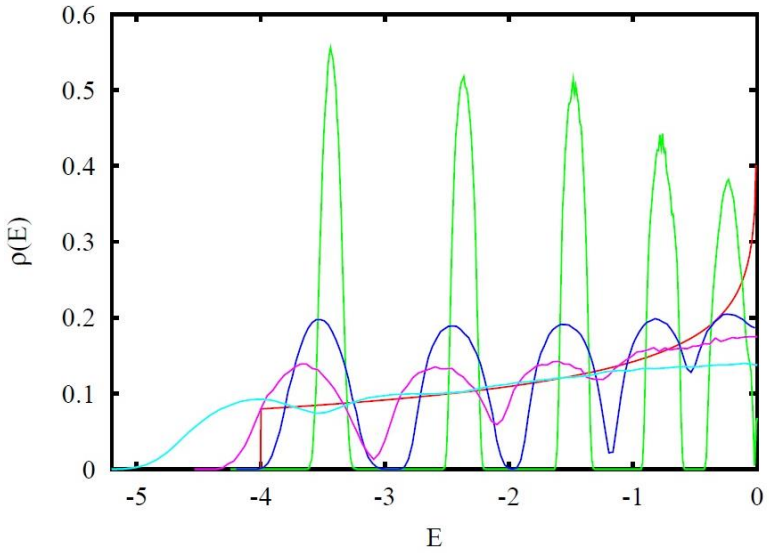

Fig.2. Density of states $\rho(E)$ of two-dimensional electron gas in a quantized magnetic field of the magnitude α=0.1 for various disorder degree W: 1 – 0.7; 2 – 2.0; 3 – 3.0; 4 - 5.0. The size of the square sample is L x L = 50 x 50. The results are obtained after ensemble averaging over 200 realizations. The density of states of the tight-binding model (W=0) is also shown by a non-oscillating line

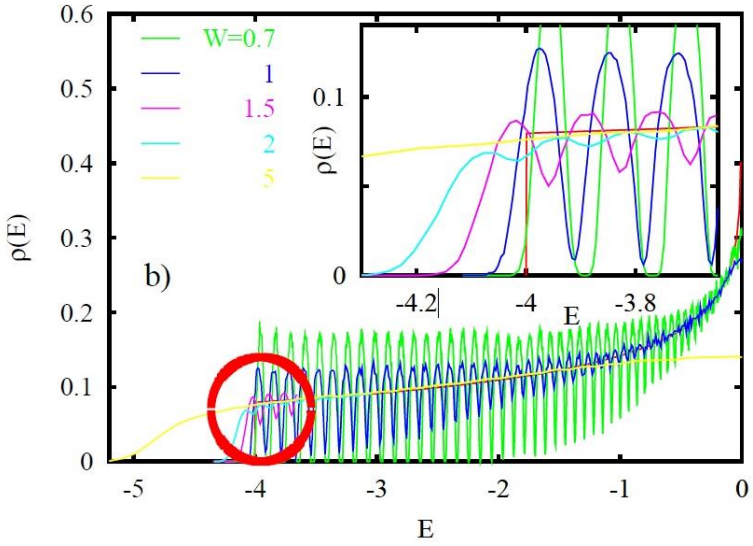

Fig.3. Density of states $\rho(E)$ of two-dimensional electron gas in a quantized magnetic field of the magnitude α=0.01 for various disorder degree W: 1 – 0.7; 2 – 1.0; 3 – 1.5; 4 - 2.0; 5 – 5.0. The size of the square sample is L x L = 50 x 50. The results are obtained after ensemble averaging over 200 realizations. The density of states of the tight-binding model (W=0) is also shown by a non-oscillating line. The inset shows the enlarged area of the density of states marked by the red circle

The localization length

The proposed numerical technique for calculation of the localization of the electronic states is based on the evaluating the Green's functions $g_E(L)$ within the single particle approximation. Here the electron transport occurs with energy E in a bar-shaped disordered system of a length L and of a finite width M. Our system should be in a thermodynamic equilibrium. For that it is connected to the two semi-infinite perfect leads (thermal baths).

The localization length Λ can be obtained in the quasi-1D limit of a long 2D stripe with the length L>>1, using the Oseledec's theorem. Then one has the following definition:

$$\frac{1}{\Lambda(W,E)} = \lim_{L\to\infty} \frac{\ln |\, g_E(L)\,|}{L}, \tag{7}$$

Thus, the inverse localization length is simply the exponential decay rate of the spatial extension of $g_E(L)$. In practice, the study of the Green's functions is mapped into the equivalent transfer-matrix calculations. The latter yields the smallest positive Lyapunov exponent identified as Λ^{-1}. Due to the convergence process of the underlying iteration procedure in the limit of large L, the statistical quantity Λ becomes self-averaging [19]. We use the transfer-matrix method developed in the paper [18], which has been successfully applied for three-dimensional disordered systems subject under high magnetic fields. In contrast to the method of calculation of the density of states performed in the previous section, which is based on the eigenvalue solvers, here we apply the scattering approach for computing the transport of the electron waves through the disordered region. While the quest for the eigenvalues requires the square geometry of the system (i.e. finite in both directions), the transfer matrix method operates on a single spatial scale, namely on the width L of the quasi-one-dimensional stripe, whose length tends to infinity. This approach provides more convenient conditions for the thermodynamic limit, rather than direct diagonalisation technique.

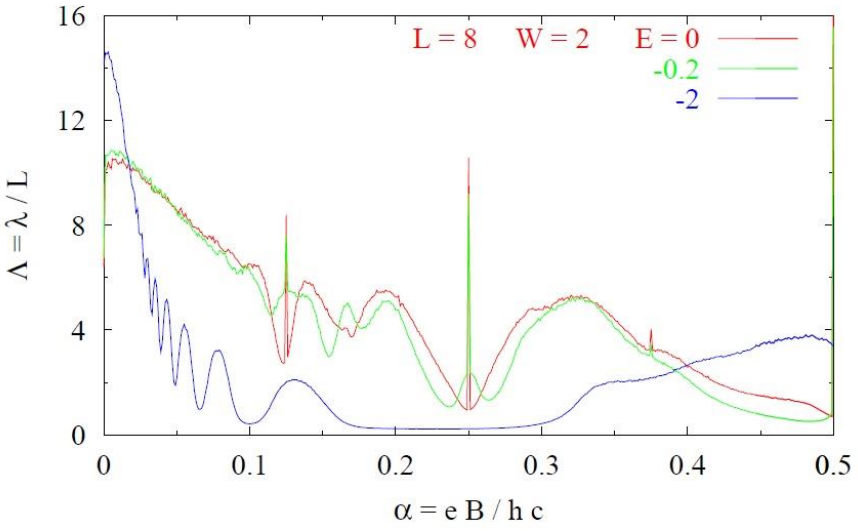

Fig.4. Renormalized localization length Λ as a function of a magnetic field α at the disorder degree $W = 2$ for various energy $E = 0$; -0.2; 2.0. Data correspond to the energy $E = 0$ for size L x L = 8 x 8 of a two-dimensional disordered system

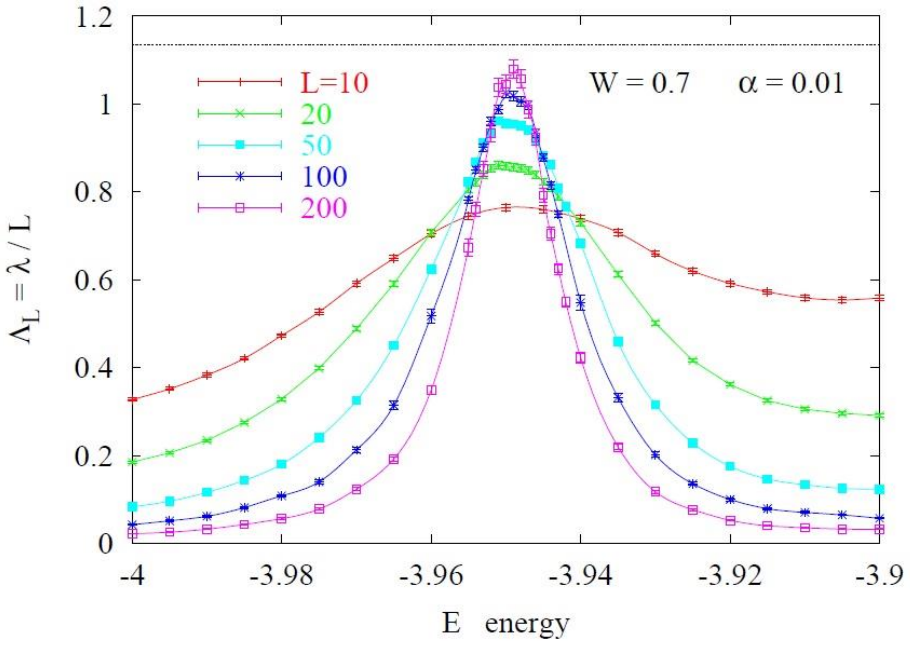

Fig.5. Renormalized localization length Λ as a function of the electron energy E in a fixed magnetic field α=0.001 in the vicinity of the lowest Landau level at the disorder degree $W = 0.7$ for various system size L : 1 – 10, 2 – 20, 3 – 50, 4 – 100, 5 - 200

All the data for the reduced localization length Λ obtained by the transfer-matrix-method have accuracy between 0.1% and 0.3% for the size L ranging from 20 up to 200. Further increase of the system size above $L = 400$ requires an improvement of the statistical accuracy of the raw data. In fact, the computing time increases as L^5/ε^2, where $\varepsilon = k\,\delta\Lambda_L/\Lambda_L$ is the relative statistical accuracy. The coefficient of proportionality k depends on the type of the boundary conditions and on computing details (that are the efficiency of the computer and the optimization of the algorithm).

In this paper we have proposed the numerical method for the calculation of the density of states and the localization length. The first raw results have been obtained for different sizes and the magnetic field. Although the preliminary data are given without detailed analysis and physical discussion, these will be provided elsewhere. Concerning the level statistics we show that the compressability of the electron spectrum is connected to the multifractal properties of the wave

functions [20]. We shift these and other relations to the problem of the quantum Hall effect for the nearest future.

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И.Х. Жәрекешев

Кванттілетін магнит өрістердегі электрондық күйлерінің тығыздығы және екіөлшемді реттелмеген жүйелерінің локализациясы

Біз біртекті магнит өрісі бар және кездейсоқ түйінді потенциалы бар екіөлшемді торда қозғалатын бір бірімен әсер етпейтін электрондарды зерттейміз. Электрондық локализациясы және күйлерінің тығыздығы трасфер-матрица және диагонализация әдісімен зерттеледі. Компьютерлік модельдеу үшін диагоналдық реттелмеген Андо моделі пайдаланылды. Зерттеліп отырған жүйе мен магнит өрісі шамасының әртүрлі өлшемдері үшін алғашқы алдын ала мәліметтер алынды. Локализация ұзындығы Шубников-де-Газ осцилляцияларын айқындайтындығы табылды. Электрондық күйлердің тығыздығы энергетикалық саңылаулармен бөлінген Ландаудың бірнеше зоналарын көрсетеді. Қоспа потенциалдың реттелмеуі өскен сайын Ландау зоналарының ені үлкейе түседі және олар өзара кайта жабыла бастайды. Қол жеткізілген қортындылардың Холлдың толықесепті кванттық эффектісіне пайдалануға болатындығы талқыланған.

Түйін сөздер: электрондық өткізгіштік, критикалық құбылыс, екі өлшемдегі электрондық газ, Холлдың кванттық эффектісі, электрондық локализациясы.

И.Х. Жарекешев

Плотность электронных состояний и локализация двумерных неупорядоченных систем в квантующих магнитных полях

Мы исследуем невзаимодействующие электроны, движущиеся в двумерной решетке с однородным магнитным полем и случайным узельным потенциалом. Изучаются электронная локализация и плотность состояний методом трасфер-матриц и методом прямой диагонализациии. Для компьютерного моделирования использовалась модель Андо с диагональным беспорядком. Первые предварительные данные получены для разных размеров исследуемой системы и величин магнитного поля. Найдено, что длина локализации проявляет осцилляции Шубникова-де-Газа. Плотность электронных состояний показывает несколько зон Ландау, разделенных энергетическими щелями. С увеличением беспорядка примесного потенциала ширина зон Ландау становится больше, и они начинают перекрываться между собой. Обсуждается применимость полученных результатов к целочисленному квантовому эффекту Холла.

Ключевые слова: электронная проводимость, критические явления, двумерный электронный газ, квантовый эффект Холла, электронная локализация