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NON-PERTURBATIVE QUANTIZATION À LA HEISENBERG: ZERO FLUX TUBE BETWEEN QUARK AND QUARK

Non-Abelian version of field distribution between two positive (negative) charges is considered. Using a two-equation approximation in the non-perturbative quantization à la Heisenberg, a flux tube stretched between two quarks (antiquarks) located at $\pm\infty$ is obtained. The dual Meissner effect is demonstrated by confining of color fields into the tube by a condensate of coset non-Abeian fields. A special case is considered when the longitudinal electric field produced by a quark located at $+\infty$ is equal and oppositely directed to the field generated by a quark located at $-\infty$ that leads to zero total electric field. We show that applying the two-equation approximation in the non-perturbative quantization à la Heisenberg for QCD one can obtain the flux tube stretched between quark and quark (antiquark and antiquark) located at $\pm\infty$ with zero longitudinal color electric field. It is shown that all color electric and magnetic fields are expelled by the scalar field that describes a condensate of coset non-Abelian fields. This effect is the analog of the Meissner effect in superconductivity for non-Abelian color fields.

Key words: non-perturbative quantization, quantum chromodynamics, two-equation approximation, flux tube.

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Гейзенберг бойынша пертурбативтік емес кванттау: кварк және кварк арасынағы нөлдік өріспен ағындық түтік

Екі оң (теріс) зарядтар арасындағы өрістің таралуының абелдік емес нұсқасы қарастырылған. Гейзенберг бойынша пертурбативтік емес кванттаудағы екі теңдеудің жуықтауын пайдаланып, $\pm\infty$ орналасқан екі кварктар (антикварктар) арасында созылған, ағындық түтік алынды. Түсті электрлік және магниттік өрістер абелдік емес өрістердің coset конденсатпен түтікке итерлетіндігімен қорытындыланатын, алынған шешімде Мейсснер эффектісінің дуалды екендігі көрсетілді. $+\infty$ орналасқан, кварктан (антикварктан) пайда болатын түсті бойлық электрлік өріс, $-\infty$ орналасқан кварктан (антикварктан) пайда болатын өріске тең, бірақ сол өріске қарама-қарсы бағытталатын жеке жағдайы қарастырылған. Гейзенберг бойынша пертурбативтік емес кванттаудағы екі теңдеудің жуықтауын пайдаланып, бір-бірінен шексіз алыс орналасқан, кварк—кварк, немесе антикварк—антикварк жұптары арасында түсті түтік алуға болатындығы көрсетілді. Сандық есептеулерді пайдаланып, түсті абелдік емес өрістер, абелдік емес өрістердің coset конденсатымен сипаттайтын, қандай да бір скаляр өріспен итерлетіндігі көрсетілді. Бұл эффект кванттық хромодинамика үшін асқын өткізгіштікте Мейсснер эффектісінің дуалды аналогы болып табылады.

Түйін сөздер: пертурбативтік емес кванттау, кванттық хромодинамика, екі теңдеудің жуықтауы, ағындық түтік.

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Непертурбативное квантование по Гейзенбергу: потоковая трубка с нулевым полем между кварком и кварком

Рассматривается неабелева версия распределения поля между двумя положительными (отрицательными) зарядами. Используя приближение двух уравнений в непертурбативном квантовании по Гейзенбергу, получена потоковая трубка, натянутая между двумя кварками (антикварками) расположенными на $\pm\infty$. Показано, что в полученном решении имеется дуальный эффект Мейсснера, заключающийся в том, что цветные электрически и магнитные поля выталкиваются в трубку coset конденсатом неабелевых полей. Рассмотрен частный случай, когда цветное продольное электрическое поле, создаваемое кварком (антикварком), расположенным на $+\infty$ равно, но противоположно направлено такому же полю, создаваемому кварком (антикварком), расположенным на $-\infty$. Показано, что, используя приближение двух уравнений в непертурбативном квантовании по Гейзенбергу, можно получить цветную потоковую трубку между парой кварк—кварк, или антикварк—антикварк, расположенными бесконечно далеко друг от друга. Используя численные расчеты показано, что цветные неабелевы поля выталкиваются неким скалярными полем, описывающим конденсат coset неабелевых полей. Это эффект является дуальным аналогом эффекта Мейсснера в сверхпроводимости для квантовой хромодинамики.

Ключевые слова: непертурбативное квантование, квантовая хромодинамика, приближение двух уравнений, потоковая трубка.

Introduction

One of unsolved problems in quantum chromodynamics (QCD) is the problem of field distribution between quark and antiquark. Similar problem is easily solved in electrodynamics: the distribution of electric field between positive and negative charges can be easily found since Maxwell's electrodynamics is a linear theory. In QCD the problem is that the calculations should be done for non-perturbatively quantized fields because Yang-Mills theories are strongly nonlinear ones. The standard point of view is that in QCD there is the dual Meissner effect: longitudinal electric field lines get compressed to a flux tube.

The flux tube field distribution is investigated within the framework of lattice QCD. In Ref. [1], the Abelian color flux of two- and three-quark systems in the maximally Abelian gauge in lattice QCD with dynamical fermions is investigated. In Refs. [2] and [3], the non-Abelian dual Meissner effect in the SU(3) Yang-Mills theory is investigated by measuring the chromoelectric flux created by an quark-antiquark source. Lattice calculations strongly support the idea of the dual Meissner effect in QCD. However, for a more complete understanding of the nature of confinement, it is necessary to have at least approximate analytical calculations confirming this point of view.

In Ref. [4], we have shown that applying the non-perturbative quantization à la Heisenberg for QCD and using the two-equation approximation, a solution describing the flux tube between quark and antiquark located at $\pm\infty$ can be obtained. The solution is characterized by a longitudinal color electric field directed from quark to antiquark. All fields in this solution are expelled by a condensate of coset gauge fields into the flux tube. This is a non-Abelian version of the field distribution between positive and negative charges in Maxwell's electrodynamics. Here we want to consider a non-Abelian version of the field distribution between charges with the same sign in Maxwell's electrodynamics. We expect that in this case we will have two longitudinal electric fields directed oppositely. We will consider some special case when these fields are the same that leads to zero longitudinal color electric field in the flux tube.

The main idea

In Ref.[4] we have shown that in the two-equation approximation for QCD one can obtain an infinite flux tube filled with a longitudinal colour electric field and stretched between quark/antiquark located at $\pm\infty$. Here we want to show that within this approximation it is possible to obtain an infinite flux tube filled with two longitudinal colour electric fields directed oppositely.

We start with the two-equation approximation obtained in Ref. [4] and applied for the flux tube with one color longitudinal electric field. The set of equations describing such a situation is

$$\widetilde{D}_\nu F^{a\mu\nu} - [(m^2)^{ab\mu\nu} - (\mu^2)^{ab\mu\nu}]A^b_\nu = 0, \quad (1)$$

$$\phi - \left(m^2_\phi\right)^{ab\mu\nu} A^a_\nu A^b_\mu \phi - \lambda\phi(M^2 - \phi^2) = 0, \quad (2)$$

where

$$(m^2)^{ab\mu\nu} = -g^2\left[f^{abc}f^{cpq}G^{pq\mu\nu} - f^{amn}f^{bnp}\left(\eta^{\mu\nu}G^{mp\ \alpha}_{\alpha} - G^{mp\nu\mu}\right)\right], \quad (3)$$

$$(\mu^2)^{ab\mu\nu} = -g^2\left(f^{abc}f^{cde}G^{de\mu\nu} + \eta^{\mu\nu}f^{adc}f^{cbe}G^{de\,\alpha}_{\alpha} + f^{aec}f^{cdb}G^{edv\mu}\right), \tag{4}$$

$$\left(m^2_{\phi}\right)^{ab\mu\nu} = g^2 f^{amn}f^{bnp}\frac{G^{mp\mu\nu} - \eta^{\mu\nu}G^{mp\alpha}_{\alpha}}{G^{mm\alpha}_{\alpha}}. \tag{5}$$

2-point Green functions for the gauge fields $\delta\hat{A}^a_\mu \in SU(2)\times U(1)$ and for the coset $\hat{A}^m_\mu \in SU(3)/(SU(2)\times U(1))$ are defined as

$$G^{mn\mu\nu}(y,x) = \left\langle \hat{A}^{m\mu}(y)\hat{A}^{n\nu}(x)\right\rangle, \tag{6}$$

$$G^{ab\mu\nu}(y,x) = \left\langle \delta\hat{A}^{a\mu}(y)\delta\hat{A}^{b\nu}(x)\right\rangle, \tag{7}$$

where $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu$ is the field strength; $a,b,c,d =$ either 1,2,3 or 2,5,7 are the $SU(2)$ colour indices; $m,n =$ either $4,5,\cdots,8$ or 1,3,4,6,8; g is the coupling constant; f^{ABC} are the structure constants for the $SU(3)$ gauge group; $A,B,C = 1,2,\cdots,8$. The equation (1) describes $SU(2)\in SU(3)$ degrees of freedom that have non-zero expectation values, and equation (2) describes coset $SU(3)/SU(2)$ degrees of freedom with zero expectation values:

$$\hat{A}^{a\mu} = \left\langle \hat{A}^{a\mu}\right\rangle + i\delta\hat{A}^{a\mu}, \tag{8}$$

$$\left\langle \hat{A}^{m\mu}\right\rangle = 0. \tag{9}$$

We seek a cylindrically symmetric solution of equations (1) and (2) in the subgroup $SU(2)\in SU(3)$ spanned on either $\lambda^{1,2,3}$ or $\lambda^{2,5,7}$ in the form

$$A^{1,2}_t(\rho) = \frac{f(\rho)}{g}, A^{1,2}_z(\rho) = \frac{u(\rho)}{g}, \tag{10}$$

$$A^{2,5}_t(\rho) = \frac{w(\rho)}{g}, A^{2,5}_z(\rho) = \frac{v(\rho)}{g}, \tag{11}$$

$$\phi(\rho) = \frac{\phi(\rho)}{g}. \tag{12}$$

Here the first superscript indices are for $\lambda^{1,2,3}$ and the second ones – for $\lambda^{2,5,7}$. We work in a cylindrical coordinate system z,ρ,φ and the corresponding colour electric and magnetic fields are then

$$E^{3,7}_z(\rho) = F^{3,7}_{tz} = \left(E^{3,7}_z\right)_1 - \left(E^{3,7}_z\right)_2 = \frac{fv - wu}{g}, \tag{13}$$

$$E^{1,2}_\rho(\rho) = F^{1,2}_{t\rho} = -\frac{f\prime(\rho)}{g}, E^{2,5}_\rho(\rho) = F^{2,5}_{t\rho} = -\frac{w\prime(\rho)}{g}, \tag{14}$$

$$H^{2,5}_\varphi(\rho) = \varepsilon_{\varphi\rho z}F^{2,5\,\rho z} = -\frac{v'(\rho)}{g},$$

$$H^{1,2}_\varphi(\rho) = \varepsilon_{\varphi\rho z}F^{1,2\,\rho z} = -\frac{u\prime(\rho)}{g}, \tag{15}$$

where

$$\left(E^{3,7}_z\right)_1 = A^{1,2}_t A^{2,5}_z = \frac{fv}{g}, \left(E^{3,7}_z\right)_2 = A^{2,5}_t A^{1,2}_z = -\frac{wu}{g}.$$

For simplicity, we consider the case with

$$w = f, u = v. \tag{16}$$

In both cases we have the following set of equations (for details see Appendix 4)

$$-f'' - \frac{f\prime}{\rho} + m^2\phi^2 f = \mu^2 f, \tag{17}$$

$$-v'' - \frac{v\prime}{\rho} + m^2\phi^2 v = \mu^2 v, \tag{18}$$

$$\phi'' + \frac{\phi\prime}{\rho} = \phi[\tilde{\alpha}(-f^2 + v^2) + \lambda(\phi^2 - M^2)]. \tag{19}$$

We see that equations (17) and (18) are Schrödinger-type equations with a solution $v(\rho) = kf(\rho)$, where k is a constant, ϕ is the potential and μ is an eigenvalue. In this case we can rewrite the set of equations (17)-(19) as follows

$$-f'' - \frac{f\prime}{x} + \phi^2 f = \mu^2 f, \tag{20}$$

$$\phi'' + \frac{\phi\prime}{x} = \phi[\alpha f^2 + \lambda(\phi^2 - M^2)]. \tag{21}$$

Here $\alpha = \tilde{\alpha}(k^2 - 1)$ and it can be an arbitrary real number; we redefined $m\phi/f(0) \to \phi$, $\lambda/$

$m^2 \to \lambda$, $mM \to M$, $f/f(0) \to f$, $x = \rho f(0)$. Numerical investigation shows that regular solution to (20) and (21) does exist only for some positive $\alpha > 0$. It is necessary to note that because of (16) the total longitudinal electric field $E_z^{3,7} = 0$, and this leads to the fact that equations (17), (18), and (20) do not have non-linear terms like $f v^2$.

The results of numerical calculations are presented in Figs. 1 and 2. We see that we have the dual Meissner effect: the longitudinal electric fields $\left(E_z^{3,7}\right)_1$ and $\left(E_z^{3,7}\right)_2$ are confined into a tube by the scalar field ϕ (which is a condensate of the coset fields).

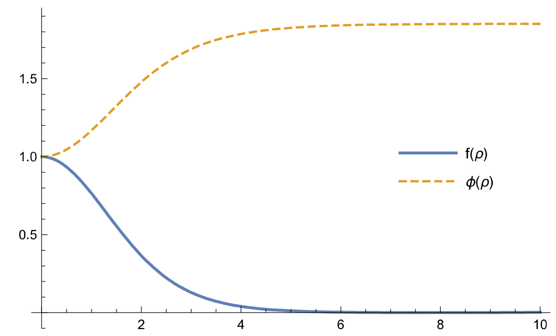

Figure 1 – The profile of the functions $f(\rho), \phi(\rho)$

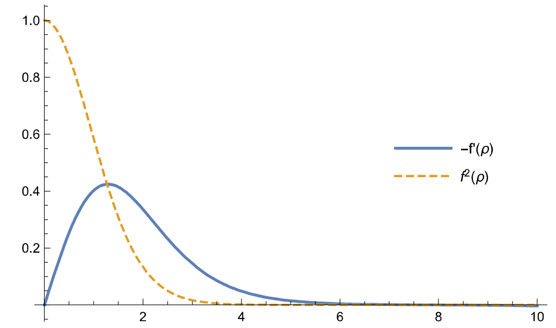

Figure 2 – The profiles of electric and magnetic fields $gE_\rho^{1,2}(\rho) = gE_\rho^{2,5}(\rho) = -f'(\rho)$, $gH_\varphi^{2,5}(\rho) = gH_\varphi^{1,2}(\rho) = -f'(\rho)$, $g\left(E_z^{3,7}\right)_1 = -g\left(E_z^{3,7}\right)_2 = f^2(\rho)$

Discussion and conclusions

We have shown that applying the two-equation approximation in the non-perturbative quantization à la Heisenberg for QCD one can obtain the flux tube stretched between quark and quark (antiquark and antiquark) located at $\pm\infty$ with zero longitudinal color electric field. It is shown that all color electric and magnetic fields are expelled by the scalar field that describes a condensate of coset non-Abelian fields. This effect is the analog of the Meissner effect in superconductivity for non-Abelian color fields.

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Appendix

Coefficients of equations (17)-(19)

$SU(2)$ subgroup spanned on $\lambda^{1,2,3}$

We use the following Ansätze for the 2-point Green functions $G^{ab\mu\nu}$

$$G^{ab\mu\nu}(y,x) \approx \Delta^{ab}\mathcal{B}^\mu\mathcal{B}^\nu, a,b = 1,2,3; \quad (22)$$

$$C^{mn\mu\nu} \approx \delta^{mn}\mathcal{A}^\mu\mathcal{A}^\nu\phi^2, m,n = 4,5,6,7, \quad (23)$$

where $\mathcal{B}_\mu\mathcal{B}^\mu$ and $\mathcal{A}_\mu\mathcal{A}^\mu$ are constants and

$$\Delta^{ab} = \mathrm{diag}(\delta_1, \delta_2, \delta_3, 0,0,0,0,0), \quad (24)$$

$$\delta^{mn} = \mathrm{diag}(0,0,0, \Delta_4, \Delta_5, \Delta_6, \Delta_7, 0), \quad (25)$$

$$\mathcal{A}^\mu = \left(0,0,\mathcal{A}_\rho, \frac{\mathcal{A}_\varphi}{\rho}\right), \quad (26)$$

$$\mathcal{B}^\mu = \left(0,0,\mathcal{B}_\rho, \frac{\mathcal{B}_\varphi}{\rho}\right). \quad (27)$$

We choose $\mathcal{B}^\mu$ and $\mathcal{A}^\mu$ in such form because $A^{\cdots}_{t,z}$ from (10) and (11) are non-zero: $A^{\cdots}_{t,z} \neq 0$. Substitution of (24)-(27) in (22) and (23) gives us [for $u(\rho) = v(\rho), w(\rho) = f(\rho)$]:

$$\mu_1^2 = g^2\left(\mathcal{B}_\rho^2 + \mathcal{B}_\varphi^2\right)(\delta_2 + \delta_3), \quad (28)$$

$$\mu_2^2 = g^2\left(\mathcal{B}_\rho^2 + \mathcal{B}_\varphi^2\right)(\delta_1 + \delta_3), \quad (29)$$

$$m^2 = \frac{3}{4}g^2\left(\mathcal{A}_\rho^2 + \mathcal{A}_\varphi^2\right)(\Delta_4 + \Delta_5 + \Delta_6 + \Delta_7)\phi^2, \quad (30)$$

$$\left(m_\phi^2\right)^{ab\mu\nu} A_\nu^a A_\mu^b = \frac{g^2}{2}\left(f^2 - v^2\right). \quad (31)$$

We set $\delta_2 = \delta_1$ and then

$$\mu_2^2 = \mu_1^2. \quad (32)$$

$SU(2)$ subgroup spanned on $\lambda^{2,5,7}$

Similar construction can be done for the $SU(2)$ group spanned on $\lambda^{2,5,7}$:

$$\Delta^{ab} = \text{diag}(0, \delta_2, 0,0, \delta_5, 0, \delta_7, 0), \quad (33)$$

$$\delta^{mn} = \text{diag}(\Delta_1, 0, \Delta_3, \Delta_4, 0, \Delta_6, 0, \Delta_8), \quad (34)$$

$$\mathcal{A}^\mu = \left(0,0, \mathcal{A}_\rho, \frac{\mathcal{A}_\varphi}{\rho}\right), \quad (35)$$

$$\mathcal{B}^\mu = \left(0,0, \mathcal{B}_\rho, \frac{\mathcal{B}_\varphi}{\rho}\right). \quad (36)$$

Substitution of (33)-(36) in (22) and (23) gives us [for $u(\rho) = v(\rho), w(\rho) = f(\rho)$]:

$$\mu_1^2 = \frac{g^2}{4}\left(\mathcal{B}_\rho^2 + \mathcal{B}_\varphi^2\right)(\delta_5 + \delta_7), \quad (37)$$

$$\mu_2^2 = \frac{g^2}{4}\left(\mathcal{B}_\rho^2 + \mathcal{B}_\varphi^2\right)(\delta_2 + \delta_7), \quad (38)$$

$$m_1^2 = \frac{3}{4} g^2\left(\mathcal{A}_\rho^2 + \mathcal{A}_\varphi^2\right)(4\Delta_1 + 4\Delta_3 + \Delta_4 + \Delta_6), \quad (39)$$

$$m_2^2 = \frac{3}{4} g^2\left(\mathcal{A}_\rho^2 + \mathcal{A}_\varphi^2\right)(\Delta_1 + \Delta_3 + 4\Delta_4 + \Delta_6 + 3\Delta_8), \quad (40)$$

$$\left(m_\phi^2\right)^{ab\mu\nu} A_\nu^a A_\mu^b = \frac{g^2}{4} \frac{5\Delta_1 + 5\Delta_3 + 5\Delta_4 + 2\Delta_6 + 3\Delta_8}{\Delta_1 + \Delta_3 + \Delta_4 + \Delta_6 + \Delta_8}\left(f^2 - v^2\right). \quad (41)$$

We set $\delta_5 = \delta_2$ and $4\Delta_1 + 4\Delta_3 + \Delta_4 + \Delta_6 = \Delta_1 + \Delta_3 + 4\Delta_4 + \Delta_6 + 3\Delta_8$ and then

$$\mu_2^2 = \mu_2^2, m_2^2 = m_1^2. \quad (42)$$

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