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NONLINEAR EQUATION OF QUARK-GLUON CASCADE

On the basis of experimental data on the structural functions of hadrons using the method of Poincare sections we introduce the nonlinear equation of quark-gluon cascade via recurrence relation taking into account merge processes of quarks and gluons. Introduced a discrete map is based on the hypothesis of self-similarity of the evolution of the quark-gluon structure of hadron and the evolution operator are the distribution of quarks and gluons. It is speculated that stochastic quantum fluctuations in strongly correlated quark-gluon system describes the so-called deterministic chaotic dynamics. Carried out fractal analysis of emerging structures (attractors), which stability is determined by Lyapunov exponents. The formation of stable structures in nonlinear quark-gluon evolution, apparently, is connected with the mechanism of hadronization.

Key words: quark, gluon, chromodynamics, nonlinear quantum evolution, stochasticity, fractal, self-similarity.

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Кварк-глюонды каскадтың бейсызық теңдеуі

Адронның құрылымдық функциясы бойынша тәжірибелік мәліметтерден кварк-глюонды сәйкестендіру процесін ескеріп, рекуррентті қатынас арқылы Пуанкаре қимасы әдісін қолдана отырып кварк-глюонды каскадтың сызықты емес теңдеуін енгіземіз. Енгізілген дискретті көрініс адрон квар-глюонды құрылымының өз-өзіне ұқсас эволюция гипотезасына негізделген және ле эволюция операторы кварктар мен глюондардың үлестірілуі болып табылады. Күшті қатынастағы кварк-глюонды жүйеде квантты стохастикалық флуктуация детерминді хаосты динамикамен сипатталады деп болжау жүргізіледі. Пайда болатын құрылымдарға (аттрактор-ларға) фракталды талдау жүргізілді, яғни оның орнықтылығы Ляпунов көрсеткіштерімен анықталады. Сызықты емес кварк-глюонды эволюцияда орнықты құрылымның қалыптасуы адрондалу механизмімен байланысты болуы мүмкін.

Түйін сөздер: кварк, хромодинамика, сызықты емес квантты эволюция, стохастикалық, фрактал, өзіұқсастық.

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Нелинейное уравнение кварк-глюонного каскада

Исходя из экспериментальных данных по структурным функциям адрона, используя метод сечений Пуанкаре мы вводим нелинейное уравнение кварк-глюонного каскада через рекуррентные соотношения с учётом процессов кварк-глюонных слияний. Введённое дискрет-ное отображение основано на гипотезе само-подобия эволюции кварк-глюонной структуры адрона и оператором эволюции являются распределения кварков и глюонов. Предпологается, что квантовые стохастические флуктуации в сильно коррелированной кварк-глюонной системе описываются так называемой детерминированной хаотической динамикой. Проведён фрактальный анализ возникающих структур (аттракторов), устойчивость которых определяется показателями Ляпунова. Формирование устойчивых структур в нелинейной кварк-глюонной эволюции, по-видимому, связано механизмом адронизации.

Ключевые слова: кварк, хромодинамика, нелинейная квантовая эволюция, стохастичность, фрактал, само-подобие.

Introduction

Consideration of the contribution to the quark-gluon distribution of bremsstrahlung of gluons leads to a violation of Bjorken's scaling and is determined by known linear evolution equations**:** Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) [1-3], Balitsky-Fadin-Kuraev-Lipatov (BFKL) [4-5] and Gribov-Levin-Ryskin-Mueller-Qiu (GLR-MQ) [6-7]. Proposed many ways to the modeling of evolution equations with non-perturbative nonlinearities, considering the gluon recombination. In addition to the gluon splitting functions, the nonlinear gluon recombination processes become important. The action Yang-Mills (Y-M) [8] already contains cubic and quartic nonlinear interaction terms in the field strength tensor:

$$S_{Y-M} = -\frac{1}{4}\int d^4x\, F^a_{\mu\nu}(x)\, F_a^{\mu\nu}(x) \qquad (1)$$

As is known, problems arise in the mathematical method of describing quantum chromodynamics at large distances, when perturbation theory for the decomposition in $\alpha_s(Q^2)$ is not applicable. Opportunities for the formation of regular structures associated with the effective competition of different types of interactions: mergers and splittings of quarks and gluons. Under the influence of the quantum fluctuations of the amplitudes of the processes there are structures which have some scale with complex self-organization. Consideration of the contribution to the quark-gluon distribution of gluons bremsstrahlung leads to a violation of Bjorken's scaling and it is determined by known linear evolution equations. There are different approaches to accounting for mergers with non-perturbative nonlinearities at the gluon recombination.

In quantum physics the processes are probabilistic in nature. According to the Feynman integrals [9]: the amplitude of the transition probabilities from one state to another is the sum of the amplitudes of all possible trajectories and is written as a functional integral:

$$\psi = \int e^{\frac{iS(x)}{\hbar}} D x(t) \qquad (2)$$

where $\hbar$ is the Planck constant, the action S(x) is an operator of quantum evolution. $\int D\,x(t)$ is a conditional entry functional integration over all trajectories x(t). Rapid oscillations in the imaginary exponent is reduced and there are only trajectories with minimal action. According to the ideas of R. Feynman, in the quantum world it is possible to speak about well-defined trajectories, only the particle does not move along the one selected trajectory, and the infinite totality. Particle can move along any trajectory and amplitude of this trajectory in response will be included with a certain weight. There are different approaches to accounting for mergers with non-perturbative nonlinearities at the gluon recombination.

Nonlinear quark-gluon cascade

Considering the evolution as a discrete quantum process we use the mathematical apparatus of mappings within the framework of nonlinear dynamics theory**.** In the spirit of Feynman's path integrals we propose [10-11] a nonlinear stochastic equation in the form of the evolution of nucleon structure function $F_2(x,Q^2)$, which represents the evolution nonlinear operator showing the distribution in the momentum representation:

$$\frac{\partial\vec{x}}{\partial t} = R\cdot F(\vec{x},t) \qquad (3)$$

Using the method of Poincare sections (choosing the share of momentum as a one-dimensional section of the phase space of partons momentum distribution) we have an evolution equation

$$x_{t+1} = R\cdot\hat{F}(x_t) \qquad (4)$$

Here Bjorken's/Feynman's variable x_t is the momentum fraction at discrete time index (t=0,1,2…**)** and R is the control parameter that characterizes the degree of coupling embossed parton with the totality of the remaining partons in the nucleon at the certain energy $\sqrt{S}$ and determines the character of observing regimes. To switch to continuous time allows the build, known as the Poincare section. In the framework of our quality approach we use the renormalization-group approach to the evolution equation, allowing to recreate a physical picture of the critical behavior. So for the quark-gluon cascade, we enter an iterative map in which a number of the quarks and gluons in (t+1)-th generation are proportional to the number of them in t-th generation. The number of partons are changing, but remains on total momenta. Thus, the probability to find a parton with a fraction of momentum **x** at time **t+1** is defined by the impulse distribution of partons in the time **t**. Positive terms of hadron structure function meet the increasing of the quarks (q) and gluons (g) number at cascade: $q \to q + g$ and $g \to g + g$ and negative terms is the reduction, i.e. quark-antiquark, quark-gluon and gluon-gluon recombination. Using the method of Poincare sections (choosing the share of momentum as a one-dimensional section of the phase space of partons momentum distribution) and considering that the evolution operator is determined by hadron structure functions (F_2), we use a one-dimensional map.

Numerical solution of the nonlinear equation

Numerical solution of the nonlinear equation has shown the existence of an evolution termination in the field of small values of parameter. Small perturbations do not change the Q-G condition (R<<1). The increase in R leads at first only to the excitation stable state. With further increase of the parameter occur repeated bifurcation (splitting) of period-doubling calculations of the quarks phase trajectories have shown the presence of the chaotic dynamics at R>>0 as a consequence of bifurcations. The scale of successive splittings of elements of limit cycles after each bifurcation is determined by

$$\alpha = Lim[x_m - x_0]/[x_{m+1} - x_0] \approx 2.5\,, \quad (5)$$

where x_m is the element of a limit cycle nearest to the element cycle x_0. In a state of dynamic chaos two close orbits in phase space diverge exponentially with time with Lyapunov's coefficient in the exponent

$$\lambda = \frac{1}{T}\ln|\mu|\,, \quad (6)$$

which in a computer simulation, is calculated using parallel running of two close initial conditions and examines their divergence. By computer simulation the studies of the formation of stable structures in quark-gluon cascade, including recombination processes. The nature of stability of fixed points (cycles) and the type of bifurcations of mappings are determined by their multipliers. In turn, multipliers are the own numbers of the Jacobian matrix perturbations. The maximum value x_{t+1} is found from $\frac{dx_{t+1}}{dx_t} = 0$. The Jacobian is

$$J = \left|\frac{dx_{t+1}}{dx_t}\right| \quad (7)$$

and the map is stable at a point x_0 if $J(x_0)<1$. When the coupling constant $\alpha_s(Q^2)$ is small, the evolution is incoherent, if the relationship is strong enough that can occur spontaneous synchronization quark-gluon movements. Dynamic quark-gluon systems are highly sensitive to the initial conditions. The calculation of the Lyapunov exponent for stationary periodic and chaotic processes is represented in Fig.1. In Fig.2 the lack of influence of small perturbations at small values of the control parameter R, the transition to the stationary mode at R=0.7, the bifurcation of the fixed point attractor at R=0.8 and the transition Q-G system into a chaotic regime at R=1.

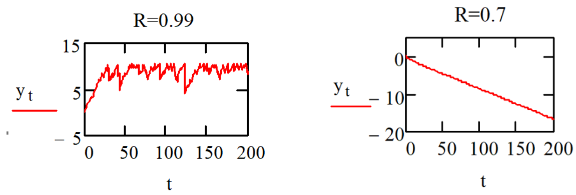

Figure 1 – Calculations of trajectories to compute the Lyapunov exponent $y_t=\ln(d_t/\varepsilon)$

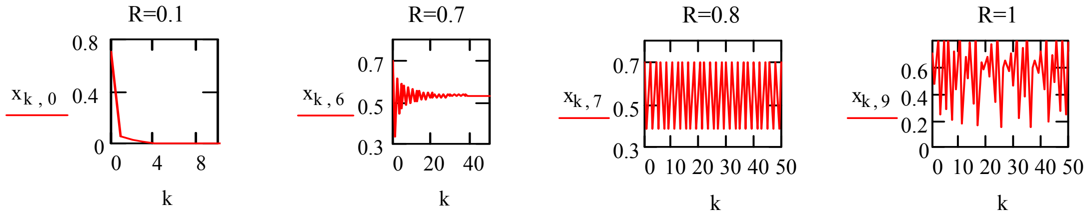

Figure 2 – Q-G evolution at different values of R

Fractal analysis

The structure of the bifurcation diagrams in Fig.3 display of SF self-similar and thus, the chaotic system has inherent properties of fractals.

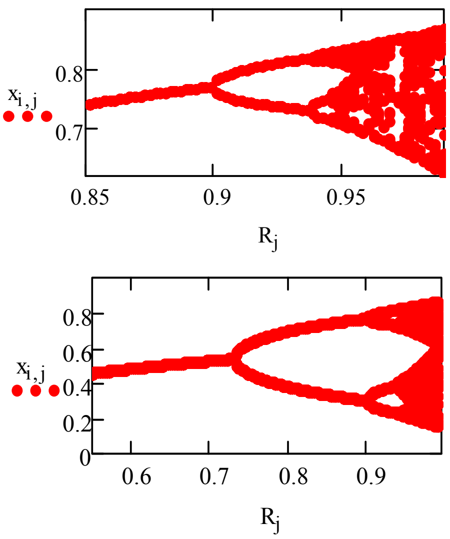

Figure 3 – Self-similarity of the bifurcation diagrams

Fractal analysis of structure functions S_m carried out the averaging over all k values, as defined as

$$S_m := \frac{1}{K-2^m} \cdot \sum_{k=0}^{K-2^m} \left| X_{k+2^m} - X_k \right| \qquad (6)$$

and is shown in Fig.4.

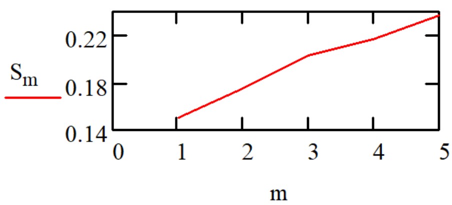

Figure 4 – Fractal analysis of the structural features of signal S_m

Isolated "Windows"

The presence of "voids" in the bifurcation diagram indicates the presence of "hadron-like phase", which is clearly seen in Fig.5

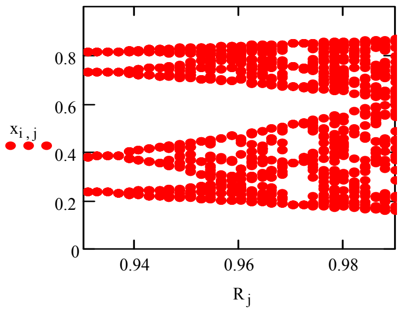

Figure 5 – The presence of "voids" in the bifurcation diagram

The controlling parameter is the energy of the collisions, the change of which leads to splitting of the phase trajectories. The exponential growth of the average multiplicity with increasing collision energy is in good agreement with the experimental data. The dependence of the control parameter R of the collision energy $\sqrt{S}$ (GeV), parameterized in the form:

$$R_n = R_\infty - \frac{a}{(S^{3/2})_n} \qquad (8)$$

For each value of R∈[0.2;R_∞] there is only one stable limit 2^n-cycle on the unit interval [0;1] and position of each element of the cycle can be calculated with a given accuracy. For energy $(\sqrt{S})_n$ in this event can be calculated the value of R_n and the corresponding 2^n cycle. The distribution of secondary particles in the momentum phase space is the images of distribution of elements of 2^n limit cycles to the unit interval. The rate of convergence of the control parameter is similar to the Feigenbaum parameter for the multifractal of the known logistic mapping:

$$\delta = \frac{R_{n+1} - R_n}{R_{n+2} - R_{n+1}} \approx 4.6$$

In Fig. 6 shows the reconstructed attractor.

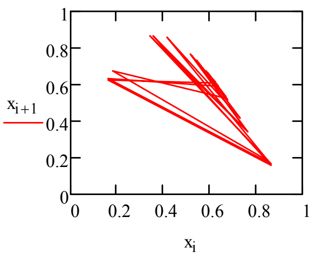

Figure 6 – Reconstruction of the attractor

Conclusion

There are nonperturbative effects associated with initial transverse momenta of partons inside the hadron and there are always fatal even quantum zero fluctuations. It is possible that a steady structure formation in nonlinear quark-gluon evolution is a mechanism of hadronization. Arising in the quark-gluon cascade the strange attractor with a fractal self-similar structure display a new nonlinear phenomenon in the hadron physics is deterministic chaotic dynamics. Self-similarity is related to the so-called power-law dependence on parameters. Dynamic quark-gluon systems are highly sensitive to the initial conditions.

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